EFFICIENT SOLUTION PROCEDURES FOR MULTISTAGE STOCHASTIC FORMULATIONS OF TWO PROBLEM CLASSES

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EFFICIENT SOLUTION PROCEDURES FOR MULTISTAGE STOCHASTIC FORMULATIONS OF TWO PROBLEM CLASSES

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To my family

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SUMMARY

Most real world optimization problems include uncertainty in the problem parameters, which are usually defined by probability distributions. Stochastic programming is used to model such problems, where the goal is to determine a policy that minimizes or maximizes the expectation of a function of the problem parameters and decision variables. However, a major drawback of stochastic programming models is that their size and complexity grow exponentially with the number of input parameters. Therefore, realistic instances of stochastic programs are large scale optimization models, for which efficient solution procedures need to be developed. We consider two classes of stochastic programming models which are motivated by two applications related to the field of aviation. These applications are used to develop models and solution procedures for two general problem classes, namely the stochastic network capacity planning problem and the project portfolio optimization problem.

The first stochastic programming problem we consider is the network capacity planning problem, which arises in capacity planning of systems with network structures, such as transportation terminals, roadways and telecommunication networks. We study this problem in the context of airport terminal capacity planning, and infer results for the general class of such problems. In the airport terminal capacity planning problem, the objective is to determine the optimal design and expansion capacities for different areas of the terminal in the presence of uncertainty in future demand levels and expansion costs, such that overall passenger delay is minimized. We model this problem as a nonlinear multistage stochastic integer program, which contains a multicommodity network flow structure representing the flow of passengers

in the terminal. The formulation requires the use of time functions for maximum delays in passageways and processing stations, for which we derive approximations that account for the transient behavior of passenger flow. The deterministic equivalent of the developed stochastic programming model is solved via a branch and bound procedure, in which a bounding heuristic is used at the nodes of the branch and bound tree to obtain integer solutions. This improved approach is shown to be significantly effective in reducing the solution time of the problem over standard approaches.

In the second study, we consider the project portfolio optimization problem. This problem falls in the class of stochastic programs in which times of uncertainty realizations are dependent on the decisions made. We study this problem within the context of optimizing aviation technology development portfolios. In this problem, the amount of investment in a given technology project determines when the uncertainty in the performance of the technology is revealed. In general, the project portfolio management problem deals with the selection of research and development (R&D) projects and determination of optimal resource allocations for the current planning period such that the expected total discounted return or a function of this expectation for all projects over an infinite time horizon is maximized, given the uncertainties and resource limitations over a planning horizon. The problem contains endogenous stochastic parameters, i.e. some parameters such as the returns are known only after making an investment. Accounting for this endogeneity, we propose efficient modeling and solution approaches for the resulting multistage stochastic integer programming model. We first develop a formulation that is amenable to scenario decomposition, and is applicable to the general class of stochastic problems with endogenous uncertainty. We then demonstrate the use of the sample average approximation method in solving large scale problems of this class, where the sample problems are solved through Lagrangian relaxation and lower bounding heuristics.

Practical and theoretical contributions of the proposed approaches to the two classes of the stochastic programming problems studied are significant. In the practical context, implementation of the models will lead to significant savings in operations, especially since no previous comprehensive mathematical models exist for the two types of problems. Theoretical contributions on the other hand include the development of novel procedures to solve the problems efficiently, as well as the development of delay time functions for transient queuing systems.

CHAPTER I

INTRODUCTION

Most decision problems involve uncertainty, and stochastic programming (SP) with recourse is a method for solving optimization problems where there is uncertainty. Dantzig (1955) was the first to introduce a recourse model where the solution could be adapted based on the outcome of a random event. Since then, the field of SP has grown and become an important tool for optimization under uncertainty.

A stochastic program results when some of the parameters in a mathematical program are described as random variables. A key assumption in SP is that probability distributions of these random parameters are known. The objective of SP is to identify a feasible policy that minimizes or maximizes the expected value of a function of decision variables and parameters over all possible realizations of the random variables.

The most widely studied SP models are two-stage models. In these problems, a decision is made at the beginning of the first stage without any certainty as to the values of the random parameters. At the beginning of the second stage, after observations regarding the uncertain parameters are made during the first stage, a recourse decision can be made to compensate for or fine tune the first-stage action. The optimal policy for a two stage model includes the best decision in the first stage considering the possible realizations of the random parameters, as well as the best recourse decision in the second stage for each possible realization.

A generalization of the two-stage problems is multistage SP models. In these models, a sequential structure exists, in which certain decisions are made at the beginning of each stage, followed by observations of the random parameters during that stage. In this study, we focus on multistage stochastic programming problems, which are some of the most difficult problems of mathematical programming as their size and complexity grow exponentially with the number of stages and the number of random parameters. We consider two such models based on two practical applications. These models differ from standard multistage approaches, since they contain additional complicating factors such as endogenous uncertainty and network structures. In the following sections, we formally define the general class of multistage stochastic programming problems, and then discuss the two areas of application which lead to the development of two large-scale multistage stochastic programming models.

1.1 Multistage Stochastic Programming Models

The general multistage stochastic program with recourse can be expressed as follows:

$$\begin{aligned}
&\min f_1(x_1) + \mathbf{E}_{\xi_1}[\min f_2(x_1, x_2, \xi_1) + \mathbf{E}_{\xi_2|\xi_1}[\min f_3(x_1, x_2, x_3, \xi_1, \xi_2) \\
&\qquad\qquad + \ldots + \mathbf{E}_{\xi_{T-1}|\xi_1,\ldots,\xi_{T-2}}[\min f_T(x_1, \ldots, x_T, \xi_1, \ldots, \xi_{T-1})] \ldots]] \qquad (1) \\
&\text{s.t.} \qquad g_1(x_1) \leq 0 \\
&\qquad\qquad g_2(x_1, x_2, \xi_1) \leq 0 \\
&\qquad\qquad \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2) \\
&\qquad\qquad g_T(x_1, \ldots, x_T, \xi_1, \ldots, \xi_{T-1}) \leq 0 \\
&\qquad\qquad x_t \in X_t \qquad\qquad\qquad t = 1, 2, \ldots, T \qquad\qquad (3)
\end{aligned}$$

where x_t and ξ_t are vectors of decision variables and random parameters, respectively, such that $x_t \in \mathbf{R}^{n_t}$ and $\xi_t \in \mathbf{R}^{d_t}$, for $t = 1, 2, \ldots, T$. Furthermore, $f_t : \mathbf{R}^{n_1+\ldots+n_t} \times \mathbf{R}^{d_1+\ldots+d_{t-1}} \rightarrow \mathbf{R}$ and $g_t : \mathbf{R}^{n_1+\ldots+n_t} \times \mathbf{R}^{d_1+\ldots+d_{t-1}} \rightarrow \mathbf{R}^{m_t}$. If all the random parameters are finitely distributed and Ω is the set of all possible combinations of realizations of random parameters, i.e. scenarios, then a probability p_ω can be associated with each scenario ω. In addition, if the objective function and the constraints are linear, then the deterministic equivalent of the multistage stochastic program (1)-(3) can be written as:

$$\min \sum_{\omega \in \Omega} p_\omega [c_1^\omega x_1^\omega \quad + c_2^\omega x_2^\omega + \ldots + c_T^\omega x_T^\omega] \tag{4}$$

$$\begin{aligned} \text{s.t.} \quad & A_{11}^\omega x_1^\omega && \leq b_1^\omega && \forall \omega \in \Omega \\ & A_{21}^\omega x_1^\omega + A_{22}^\omega x_2^\omega && \leq b_2^\omega && \forall \omega \in \Omega \\ & \vdots && \vdots && \\ & A_{T,T-1}^\omega x_{T-1}^\omega + A_{TT}^\omega x_T^\omega && \leq b_T^\omega && \forall \omega \in \Omega \end{aligned} \tag{5}$$

$$x_t^\omega - x_t^{\omega'} = 0 \qquad \forall \omega, \omega' \in \Omega : (\xi_1^\omega, \ldots, \xi_t^\omega) = (\xi_1^{\omega'}, \ldots, \xi_t^{\omega'}), \quad t = 1, 2, \ldots, T \tag{6}$$

$$x_t^\omega \geq 0 \qquad \forall \omega \in \Omega, \quad t = 1, 2, \ldots, T \tag{7}$$

In the above formulation, constraints (6), which are called the nonanticipativity constraints, ensure that the decisions made at stage t are the same for all scenarios that have the same history until stage t. The nonanticipativity constraints are necessary to honor the information structure of the problem, and they can be expressed in several different ways (Higle, 2005). Possible realizations of the random vectors ξ_t can be depicted using a tree representation, as shown in Figure 1. The leaf nodes of a scenario tree correspond to scenarios defined by the paths from the root node to the leaf nodes.

The multistage stochastic program (4)-(7) is based on a scenario formulation. Another formulation method for multistage stochastic programming problems is the node formulation, which is based on subproblems defined for the nodes of the scenario tree. If $\mathcal{N}$ is the set of nodes of the scenario tree, then decisions x_n at each node $n \in \mathcal{N}$ are restricted by decisions made at the ancestor node $a(n)$ of this node. Letting $A_n = A_{t,t-1}^k$, $A_n' = A_{tt}^k$, and $b_n = b_t^k$ for all scenarios k passing through node n at stage t, we can develop the following formulation for multistage stochastic programming

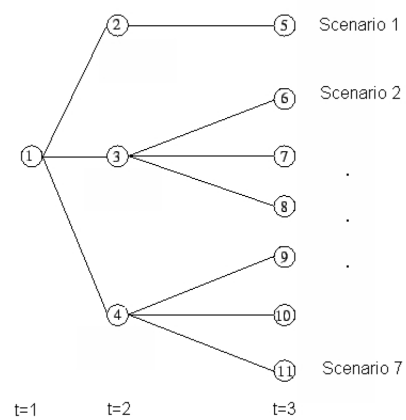

Figure 1: A scenario tree for a multistage stochastic program depicting possible realizations of the random parameters over three stages

problems:

$$\min \sum_{n \in \mathcal{N}} p_n c_n x_n \tag{8}$$

$$\text{s.t.} \quad A_n x_{a(n)} + A_n^{'} x_n \leq b_n \quad \forall i \in \mathcal{N} \tag{9}$$

$$x_n \geq 0 \quad \forall i \in \mathcal{N} \tag{10}$$

1.1.1 Solution Approaches to Multistage Stochastic Programming Problems

Solution approaches developed for multistage stochastic programming problems are mostly applicable to continuous linear multistage problems only. One such method is nested decomposition of Birge (1985), which consists of a recursive implementation of the Benders decomposition algorithm. Another decomposition approach, the progressive hedging method of Rockafellar & Wets (1991) iteratively solves a scenario subproblem and progressively enforces the nonanticipativity constraints. Rosa & Ruszczynski (1996) apply augmented Lagrangian techniques to multistage stochastic programming problems in two different ways: by decomposing the problem into scenarios and by decomposing it into nodes at each stage. However, these methods are inapplicable to multistage stochastic integer programming problems due to nonconvexities arising from the integer variables in later stages.

There are no general solution algorithms for multistage stochastic integer programming problems. Most solution approaches are problem specific, and usually exploit the scenario structures by decomposing the deterministic equivalent of the stochastic program. A dual scenario decomposition is described in Caroe & Schultz (1999), where the nonanticipativity constraints are subjected to Lagrangian relaxation. The resulting Lagrangian dual contains a separable minimization, which reduces to solving several single-scenario size integer linear problems. The solution of the dual provides a lower bound for the original primal problem, and heuristic methods are used to obtain upper bounds from the dual solution. The procedure is embedded in a branch and bound scheme, which is finite if all the decision variables are discrete. A similar decomposition scheme with some improvements is also discussed in Bruni (2005).

Due to the difficulty of calculating expected values, sampling based methods are also used in solving multistage stochastic integer programming problems. Norkin *et al.* (1998) describe a stochastic branch and bound method, which uses stochastic upper and lower estimates of the optimal value of the objective function. Stochastic upper and lower bounds are constructed by various means, such as by solving the expected value problem through Monte Carlo sampling. Another common approach utilized in obtaining near-optimal solutions to multistage stochastic programs is the sample average approximation method (Kleywegt *et al.*, 2002). In this approach, a set of samples are generated according to the probability distributions of the random parameters, and the expected value functions in the problem are estimated by a sample average function. The resulting problem is then solved using a deterministic algorithm, and the procedure is repeated until a stopping criterion is met. However, to obtain a valid upper statistical bound for the actual multistage stochastic programming problem, one needs to construct an implementable and feasible policy.

Furthermore, Shapiro (2003) shows that conditional sampling procedure must be employed to ensure that the sample average approximation provides a consistent lower bound for the corresponding multistage stochastic problem.

The two classes of multistage stochastic integer programming problems we consider are motivated by the following two applications.

1.2 Motivating Application for the Stochastic Network Capacity Planning Problem: Airport Terminal Planning

For any network, where flow rate is a function of the amount of flow on an arc, network capacity planning problem deals with the determination of optimum capacity allocations on the arcs of the network so that some function of the flow rates is minimized or maximized. This description captures traffic flow networks as well as other general queuing networks, since queues can be represented by the arcs of a network. In the stochastic version of the problem certain parameters such as the inflow rates or total available capacity are assumed to be unknown. In this study, in addition to developing a general formulation for the stochastic network capacity planning problem, we also devise special approximations for the relationship between flow rates and congestion in pedestrian flow networks and transient queuing systems. These relationships can be modified depending on the type of flow in the network.

The airport terminal planning problem is a special case of the stochastic network capacity planning problem, and the modeling and solution procedures developed for airport terminal planning are directly applicable to the generalized network capacity planning problem. Hence, we study the general solution methodologies in the context of airport terminal planning, which we describe in detail below.

Congestion is a significant problem for the hundreds of thousands of passengers flying in and out of major airports each day. This problem has been exacerbated over the last several years by the heightened levels of security. Hence, capacity planning during the airport terminal design process is more important than ever, suggesting a need for the development of more accurate analysis methods. However, the uncertainty associated with future passenger demand levels and the complexity of the airport terminals make this a difficult task.

Hamzawi (1992) emphasizes the need for a solution to the problem of congestion caused by lack of capacity, arguing that if no remedial actions are taken, it could lead to an eventual functional breakdown of the airport system. In practice, most such actions are realized in the form of costly expansion projects, because there are limited resources available during the initial construction, and great uncertainty as to future demand.

Changing demand levels and security requirements are the main reasons for expansion projects. For example, the terminal area at Los Angeles International Airport is being reconfigured to provide for security improvements by replacing parking structures with new passenger terminals that will provide additional screening capabilities. The total cost of the overall master plan is estimated to be around 11 billion dollars (LAX, 2004). Similarly, Dallas-Fort Worth International Airport has implemented a master plan to expand its current narrow passageway design by building new terminals and adding gates with an expected cost of approximately 3 billion dollars (DFW, 1997). The plan was originally developed in 1997 for a twenty year planning horizon, and was based on a demand forecast that suggested an 85% increase according to the demand at the start of the planning period. Clearly, a model that accounts for the uncertainty in these forecasts in an accurate way could lead to savings of billions of dollars in these expansion projects.

Based on the argument above, it is crucial that the need for expansion and the costs associated with the initial design and future expansion projects are minimized. Significant, long-lasting increases in airport terminal capacity can only be achieved through the building of new terminals that are designed to be expandable from their very conception. Considering that upwards of twenty airports may need to be built worldwide in the next two decades, there is a distinct need for new terminal designs that are efficient and flexible enough to accommodate the wide range of demand scenarios that are possible, given the significant, historically observed uncertainty in the demand for air transportation.

The airport terminal capacity planning problem deals with the determination of the optimal design and expansion capacities for different areas of the terminal in the presence of uncertainty with regards to future demand levels and expansion costs, such that overall passenger delay is minimized. Even when the uncertain parameters are assumed to be fixed, analytical modeling of passenger flow in airport terminals under transient demand patterns is very difficult, due to the complex structure of a terminal. Because of this difficulty, the airport terminal capacity planning problem has not been studied in a holistic fashion. Hence, studies in this area, which we discuss in Section 2.1.1, either do not account for expandability or focus only on one particular area of the terminal. In this study, we consider the airport terminal capacity planning problem as a whole, and develop methods to determine the optimal capacity requirements for each area in an airport terminal during the initial building phase, as well as the optimal expansion policy with recourse options under stochastic future demand. The optimization aims to minimize total expected passenger delay subject to budget and flow constraints.

We model this problem as a nonlinear multistage stochastic integer program, which contains a multicommodity network flow structure to model the flow of passengers in the airport terminal. The formulation requires the use of time functions for maximum delays in passageways and processing stations, for which we derive nonlinear approximations that account for the transient behavior of the passenger flow. Since no exact solution approaches exist for nonlinear multistage stochastic integer programs, we focus on the deterministic equivalent of the developed stochastic programming model. Due to the large scale of the realistic instances of the problem, standard solution approaches fail to produce good results in reasonable time. Hence, development of efficient solution algorithms is necessary to solve the problem. Any such algorithm will also be applicable to the general class of stochastic capacity planning problems, which are studied in detail for various applications in other fields, as described in Section 2.1.

As stated above, the capacity planning problem, and more specifically network capacity planning is mostly a multistage process. This is mostly due to the frequent evaluation of existing capacity and demand levels at fixed intervals according to improved forecasts and observed demand levels. For instance, capacity planning for an airport terminal is normally performed for a 15-20 year planning period with possible expansion decisions every 4-5 years. Considering the stochasticity in some of the input parameters, the resulting problem can then best be described as a multistage stochastic program.

On the other hand, a two-stage approach to the network capacity planning problem may be a more suitable method, if effects of a major change, such as a big jump in demand due to a modification in operating conditions, are to be studied. For example, Incheon International Airport in Seoul, South Korea initially opened as a two-runway airport expected to serve around 30 million passengers annually. At the end of a major development project, the airport will have four runways, hence increasing the passenger demand to an annual level of around 100 million passengers. Considering the uncertainty in these demand levels, a two-stage capacity model can be used to determine the best design capacity allocations for the airport terminal. From a modeling and computational perspective, the multistage network capacity model developed in this study can easily be modified to a two-stage problem based on the proposed framework. Hence, the developed methodology can be used for analysis of situations discussed above.

1.3 Motivating Application for the Project Portfolio Optimization Problem: Federal Aviation Administration Technology Portfolio Management

Project portfolio management involves research and development projects aimed to design, test and improve a technology, or the process of building a technology. Technology development is often an essential part of the operational strategy of an organization, during which deployment or implementation decisions are made. In most cases, organizations have several potential technologies with different characteristics that they can choose to invest in and develop using available resources. Selection of projects and allocation of the resources to the selected projects are important decisions with huge economic implications for an organization.

Characteristics of technology projects involve the resource levels required for research and development, and the projected returns after deployment, which are unknown at the time of investment, but for which some information on the uncertainty is available. Given these uncertainties and resource limitations over a planning horizon, the project portfolio management problem deals with the selection of R&D projects and determination of optimal resource allocations for the current planning period such that the expected total discounted return or a function of this expectation for all technologies over an infinite time horizon is maximized.

Federal Aviation Administration (FAA) needs to make large capital expenditures over long periods of time for the modernization of the National Airspace System (NAS). The agency has identified a set of technologies with several uncertain attributes as potential systems to invest in and develop. These uncertain attributes are defined by probability distributions, and include required investment levels, returns and the duration of implementation after the completion of technology development. A technology can be developed over multiple years, however a fixed cost is incurred for each technology project that remains active, i.e. where development has started but is not complete. After a successful development phase, the technology is deployed or implemented. It is assumed that this phase consists of only a time delay, since implementation costs are usually shared between multiple entities and are not relevant for FAA in planning technology developments. However, cost structures can also be included for situations where this assumption is not valid. Returns are accrued after the implementation or deployment is complete. Furthermore, multi-way dependencies exist between technologies, which implies that the joint return of two dependent technologies can be different from the sum of their individual returns. Given these conditions, one version of the FAA technology management problem deals with finding a process or methodology to optimize the allocation of resources to the technologies. A second version of the problem treats the future budget levels as decision variables, and assumes that certain deadlines exist to complete a subset of the technologies considered. Then the problem becomes a multi-criteria optimization problem, in which the objectives are the minimization of required budgets and maximization of the total expected return subject to the deadline constraints.

Although at first glance, it may seem that financial portfolio optimization theory could be directly applied to project portfolio management, there are clear differences between the two problems. One distinction is in the realization of returns. The realization time and the variance in the return of a technology project is dependent on the investment made on that project. However, for financial securities, both the risk and the time of return realization is independent of the amount of the security that is purchased. Assuming that no one investor will seek to make a single purchase of all or the vast majority of a company's stocks that will cause the price of the security to change by virtue of the purchase itself, the value of the security will solely be based on the performance of the company in question. A second difference between the two problems is about the correlation among project returns. In financial portfolio theory, the correlation in returns is assumed to be independent of the way in which resources are allocated. On the other hand, the correlation among the returns of technology projects is dependent on investment levels, because resources spent on one technology are taken away from other technologies, thus preventing early return realization in these technologies. Finally, a third distinction is the dependencies of technology projects in terms of produced returns. In financial theory, the cumulative return from two purchased securities is assumed to be equal to the sum of the individual returns of the securities. However, as noted above, technologies have dependencies which can have a positive or negative effect in the realization of cumulative joint returns.

No studies exist in the literature that consider the project portfolio problem in the way that it is described above. Realistic instances of this problem contain ten to fifteen planning periods with more than ten projects. Thus, the problem is a very large scale stochastic optimization model, and methods are needed to reduce the problem to one that is tractable. Although the problem is not amenable to any specific method, most tractability is assumed to be achievable by modeling the problem as a multistage stochastic program. This model, however, differs from classical stochastic programming models since the realization of the scenario tree is dependent on the decisions made. Hence, development of an efficient solution procedure that can be generalized to this special class of stochastic programming problems is a significant contribution of the study. In Chapter IV, we describe the developed general solution method and its application to the project portfolio optimization in detail, and provide examples based on the FAA portfolio management problem discussed above.

CHAPTER II

LITERATURE REVIEW

In this chapter, we describe previous work on the two motivating applications and the corresponding classes of multistage stochastic programming problems. Due to the difficulty and sizes of possible comprehensive models, studies in these areas are not many. Although some significant work has been performed on certain types of capacity planning and project portfolio optimization problems, few studies exist at the detailed level required, especially on stochastic programming problems with decision dependent scenario trees.

2.1 Previous Work on Capacity Planning Problems under Uncertainty

General capacity planning models are those in which sequential decisions on capacity expansions are made in response to the realizations of uncertain input parameters over a finite planning horizon. This problem can be formulated as a multistage stochastic program. However, introduction of fixed costs for expansion decisions add integer variables to the problem, and the solution becomes difficult even for small real-world instances.

The stochastic capacity expansion problem has been the subject of research since the 1960s. Manne (1961) models demand realizations as an infinite-horizon stochastic process, and suggests a stochastic control theory based approach to solve the problem. Giglio (1970) and Freidenfelds (1970) also study infinite-horizon problems, but none of these models are comprehensive enough to incorporate the complex operational constraints of real-world applications.

Although they suffer from the curse of dimensionality, multistage stochastic programming approaches provide more flexibility in modeling capacity planning problems. However, due to the additional complexity introduced when fixed-charge expansion costs are assumed in later stages, most approaches to the problem include two-stage models with linear cost functions. (Eppen *et al.*, 1989; Sen *et al.*, 1994; Berman *et al.*, 1994; Swaminathan, 2000; Chen *et al.*, 2002; Riis & Andersen, 2002; Riis & Lodahl, 2002; Barahona *et al.*, 2005). Of the few multiperiod models studied, Riis & Andersen (2004) use a dynamic programming algorithm to solve a multistage capacity planning problem for a telecommunications application. However, the multiperiod structure is modeled as a two-stage problem. Rajagopalan *et al.* (1998) also develop a dynamic programming based solution approach for a multiperiod model with deterministic demand, with uncertainties in the timing of capacity availability.

With increased computational power, it has become possible to consider stochastic capacity planning problems in which a fixed cost is associated with each expansion decision. However, due to the nonexistence of general solution approaches for these large-scale problems, most developed algorithms are problem specific, and exploit the problem structure in the application. For capacity planning problems, Ahmed *et al.* (2003) use a scenario tree approach to develop a multistage stochastic integer programming formulation, which is then solved using a branch and bound procedure. The branch and bound implementation incorporates tight lower bounds obtained by reformulating the problem and upper bounds obtained by a heuristic scheme. Furthermore, Ahmed & Sahinidis (2003) and Huang & Ahmed (2005) study approximation procedures for the problem, which asymptotically converge to an optimal integer solution as the number of stages increases. Another solution approach is proposed by Singh *et al.* (2005), in which variable splitting is performed to create a stronger master problem in a Dantzig-Wolfe decomposition scheme. The subproblems are single period deterministic capacity expansion problems, which can usually be solved efficiently.

Despite the existence of general capacity planning models, capacity planning for networks contains several additional complexities which require the development of different models and solution procedures. Existing literature on such problems and the need for the development of new approaches are discussed below.

2.1.1 Previous Work on Stochastic Network Capacity Planning

The network capacity planning problem, which we present using the airport terminal planning context, differs from other capacity planning models in several ways. In addition to having a nonlinear objective function, one other main difference is that the network models do not require the capacity levels to meet or exceed demand for feasibility. Instead, lack of capacity is penalized in the objective function by adding increased delay. Furthermore, it is based on a network structure, and thus contains conservation of flow constraints. Due to these complicating factors, most of the heuristic approaches described above for capacity planning are not applicable to this problem. On the other hand, any solution procedure developed for the network capacity planning problem can be applied to other capacity planning models.

Network capacity planning problems also fall in the general class of capacitated network design problems. These problems usually seek to find a network spanning a given subset of nodes, while minimizing some cost function. When studying this problem, most studies assume that point-to-point demands are deterministic. Bienstock & Gunluk (1996) and Bienstock *et al.* (1998) study this deterministic version of the problem, which is solved by cutting-plane procedures based on derived facet-defining inequalities. Magnanti *et al.* (1995) and Mirchandani (2000) discuss similar results for the related problem of network loading, in which no cost is associated with the flows. Another deterministic application for distribution networks is studied in Kalvenes & Keon (2007), where a Lagrangian based solution procedure is introduced.

Deterministic network design problems are also common in traffic network design, where the problems focus on formulations and algorithms for improvement of road capacities on an existing network. Some examples to such studies are Yang (1998) and Liu (2002), where optimal capacity allocations are obtained through deterministic discrete models. In addition, Albanese *et al.* (2003) develop a model that provides information about traffic flows and queue dynamics at the intersections of roadway junctions. However, to the best of our knowledge, stochastic traffic network design models that capture the uncertainty in flow levels and congestion do not exist in the literature.

Stochastic version of the general network design problem have been studied in Riis & Andersen (2002) and Andrade *et al.* (2004) within the context of telecommunication network design. In these studies, two-stage stochastic programming models with linear costs are described. Riis & Andersen (2004) extend this approach to the multiperiod case, but again using a two-stage model. Another two-stage stochastic model based on the Steiner tree problem is described in Gupta *et al.* (2007), where a linear programming rounding approximation algorithm is proposed as a solution procedure for two models. However, in these studies a major assumption is that the flows on the edges of the network do not interfere. This assumption is not applicable to most stochastic network design problems such as those containing congestion effects, so the development of more comprehensive models is necessary. Furthermore, none of the existing network design models consider queuing systems as part of the developed networks.

Although no network based multistage stochastic capacity planning models exist in the literature that can be applied to terminal planning, there exist general studies that discuss the capacity problem specifically at airport terminals. Most studies that consider the capacity problem are those that focus on the optimum design of airport terminals. Such studies usually include single period approaches based on short-term

	Stochastic	Multistage	Queues	Multi - commodity	Flow interference	Application/ Remarks
Albanese et al. (2003)			✓			Roadways
Bienstock&Gunluk (1996)				✓		Telecommunication networks
Bienstock et al. (1998)				✓		General
Gupta et al. (2007)	✓					General
Kalvenes&Keon (2007)					✓	Telecommunication networks
Liu (2002)						Roadways
Magnanti et al. (1995)				✓		Telecommunication networks
McCullough&Roberts (1979)			✓	✓		Passenger terminals
McKelvey (1989)			✓			Passenger terminals
Mirchandani (2000)				✓		General
Riis & Andersen (2002)	✓			✓		Telecommunication networks
Riis &Andersen (2004)	✓	Multiperiod		✓		Telecommunication networks
PROPOSED APPROACH	✓	✓	✓	✓	✓	General

Figure 2: Summary of the existing literature on network capacity planning and contributions of the proposed methodology

demand forecasts and the corresponding passenger flows within the terminal. Using this concept, Saffarzadeh & Braaksma (2000) develop a resource utilization model in which the cost of oversizing or undersizing the terminal facilities is minimized, while McCullough & Roberts (1979) present a capacity analysis model based on a study of passenger movements within the terminal during discrete time intervals. In addition, McKelvey (1989) suggests a multi-channel queuing system approach to analyze passenger processing times under different capacity levels. Although queuing models can be used for passenger flow analysis, a steady-state assumption is not valid for most passenger terminals due to the high variability in the number of arrivals and departures during a typical day. Hence, the well-known steady state results for queuing systems are inapplicable. On the other hand, transient studies are generally intractable due to the complexity of flow in most networks. Thus, most studies involve simulations to model and analyze this random and complex flow process. In these studies, simulation results are used to estimate the required capacity levels to make the operations more efficient. One such example is by Jim & Chang (1998), in which a simulation model is proposed to evaluate several terminal design alternatives.

In Figure 2, we list the existing approaches to network capacity planning, and describe the deficiencies in these studies in terms of capturing all relevant aspects of a general network capacity planning problem. We also show that the proposed methodology in this study fills these gaps in the literature through the development of a comprehensive model.

2.2 Previous Work on Stochastic Programming Problems with Decision Dependent Scenario Trees

The random parameters in a stochastic programming problem can be categorized into two types: exogenous and endogenous (Jonsbraten, 1998). Exogenous parameters are realized independent of the decisions made. For problems containing only exogenous parameters, the scenario tree is fixed, and realizations of the random parameters are assumed to occur at fixed stages defined by the scenario tree. On the other hand, endogenous parameters are those, for which the time or stage of realization depends on the decisions made. The structure of the scenario tree for these problems is not known a priori. Hence, the objective for such problems is to find optimal decisions such that when the corresponding stochastic programming problem is solved on the scenario tree defined by these optimal decisions, the solution is the best solution over all possible scenario trees.

Classical stochastic programming assumes that all random parameters of a problem are exogenous. There are very few studies on stochastic programming problems with endogenous uncertainty. Jonsbraten *et al.* (1998) is the first to address such problems, in which an algorithmic procedure to solve this type of two-stage problems is described. The proposed method includes a branch and bound scheme to determine an optimal vector of decisions, each of which has a corresponding scenario tree. Goel

& Grossmann (2004b) model the operational planning of offshore gas field developments as a multistage stochastic program with endogenous uncertainty. The stages of the problem contain decisions to install production and well platforms, which result with the realization of the uncertain parameters for the fields in which installations are performed. The problem is formulated using disjunctions, and an approximation algorithm based on decomposition and restriction of the search space is described. A similar formulation is also given in Goel & Grossmann (2004a), in which a Lagrangian duality based branch and bound procedure is proposed to solve the problem. Held & Woodruff (2005) consider a network interdiction problem where the endogenous uncertainty is in the structure of the network. Stages of the problem contains interdiction decisions followed by shortest path calculations in the interdicted network. Several problem specific heuristic solution methods are described and compared in the study. More recently, Goel & Grossmann (2006) generalize the disjunctive programming formulation in Goel & Grossmann (2004b) to problems containing both exogenous and endogenous certainty. The authors also discuss a set of theoretical properties that leads to a reduction in the problem size. However, these results are only applicable to small size problems, since they are valid only when all possible scenarios are included in the problem. Viswanath *et al.* (2004) and Tarhan & Grossman (2006) consider somewhat different versions of the above class of problems. Viswanath *et al.* (2004) address a two-stage network problem, where in the first stage survival probabilities of arcs can be changed by investment decisions. Tarhan & Grossman (2006) consider gradual uncertainty revelation over time in the synthesis of process networks. The stages of realization of the uncertain parameters are determined by decision variables, but full realization takes place in multiple stages. A disjunctive programming formulation is described, and a duality based branch and bound procedure is proposed to solve the problem.

None of the above studies contain efficient solution procedures to solve problems with endogenous uncertainty, and almost all computational studies are performed on small size problems. The general disjunctive programming formulation and the solution suggested by Goel & Grossmann (2006) does not contain a direct decomposition structure, which is typically used in solving classical stochastic programming problems. In this study, we aim to fill this gap by developing a formulation scheme that is amenable to scenario decomposition, and is applicable to the general class of such problems. In addition, effective solution procedures for the resulting subproblems are also developed.

2.2.1 Previous Work on Project Portfolio Optimization

Markowitz (1952) laid the background for modern financial portfolio theory, which has been studied extensively since then. Markowitz (1952) suggests that investors should select portfolios based on overall risk-reward characteristics of the securities, rather than investing on a single security with the best risk-reward characteristic. Tobin (1958) studies super efficient portfolios with risk free assets, while Sharpe (1964) develops the capital asset pricing models. Since then, many other modeling and optimization techniques have been proposed for financial portfolio optimization.

Although the financial portfolio theory is a well studied field, it was discussed in Section 1.3 that classical portfolio theory results can not be applied to the project portfolio optimization problem. Despite the importance and economic significance of project portfolio selection and the existence of several operations research models, the industrial use of these models has been limited. This is mainly due to the fact that none of the proposed models has been able to capture the full range of complexity that exists in technology project portfolios.

Reyck *et al.* (2005) study the impact of project portfolio management techniques on the performance of projects and portfolios of projects. The authors identify certain key components required for an effective portfolio management approach, which include the following capabilities: capturing of financial returns and risks of assets, modeling interdependencies, determination of prioritization, alignment and selection of projects, modeling organizational constraints and ability to dynamically reassess the portfolio. Linton *et al.* (2002) provide a review of proposed project portfolio management methods, and categorize the existing methods into three groups. The first category contains approaches based on net present value (NPV) calculations, while the second group consists of scoring methods and the last group covers other decision analysis tools. However, none of the considered approaches are able to model and deliver the set of capabilities identified by Reyck *et al.* (2005).

The proposed models for project portfolio management include capital budgeting models, which typically use accounting based criteria, such as return on investment or internal rate of return. These models capture interdependencies between different projects, but fail to model the uncertainty in returns and required investments (Luenberger, 1998). More recent project portfolio models capture both the uncertainty in returns and interdependencies. However, these models assume that the required cash flows for projects are known, and the investment decisions consist of binary starting or stopping decisions for projects (Ghasemzadeh *et al.*, 1999; Gustafsson & Salo, 2005). One example where the amount of resources allocated to each project is treated as a decision variable is given by Norkin *et al.* (1998). The example is formulated as a stochastic integer program, but the interdependencies between multiple projects are not modeled.

Other approaches to project portfolio management include real options based methods. Despite some disadvantages from an optimization perspective, these methods are superior to NPV based methods. Bardhan *et al.* (2006) propose a multi-period optimization model where the objective is based on real options values of the portfolio calculated according to the results from Bardhan *et al.* (2004). Campbell (2001) and Lee *et al.* (2001) model project contingencies as real options to determine optimal startup dates for the projects. Tralli (2004) devises a real options valuation architecture from a decision tree analysis structure and presents a case study. Similarly, Wu & Ong (2007) combines the mean-variance model of classical financial theory with real options, and describe a project selection methodology based on the developed framework. However, one major disadvantage of real options based approaches is that they require the estimation of cash flows for the projects. Given these estimates, these models try to determine the optimum starting, continuation or completion times for the projects in a portfolio. Thus, despite its significance, the option of rebalancing through allocation of resources in each planning period is not modeled (Cooper *et al.*, 2001). Chan *et al.* (2007) emphasize this problem and suggest a dynamic methodology based on a two-phase model of project evolution. However, the model does not capture the interdependencies or resource allocation decisions discussed above.

There are also other somewhat more simplistic approaches to the technology project portfolio problem, which either contain deterministic models or include several restrictive assumptions. Dickinson *et al.* (2001) present a model developed to optimize a portfolio of product development improvement projects. Using a dependency matrix, which quantifies the interdependencies between projects, a deterministic nonlinear integer programming model is developed to optimize project selection. April *et al.* (2003) describe a simulation optimization tool for technology project portfolio management. The tool utilizes metaheuristics to search for good technology portfolios, and is limited in capturing the interdependencies among technologies. Elfes *et al.* (2005) address the problem of determining optimal technology investment portfolios that minimize mission risk and maximize the expected science return of space missions. The solution approach described in the study is based on a deterministic linear programming formulation and sensitivity analysis. Lincoln *et al.* (2006) develop a method for prioritization of technology investments using a deterministic linear programming formulation to maximize an objective function subject to overall

	Stochastic	Inter-dependencies	Organizational constraints	Project Selection	Resource Allocation	Complete Dynamic Reassessment
April et al. (2003)	✓			✓	✓	
Bardhan et al. (2006)	✓	✓	✓	✓		
Campbell (2001)	✓	✓		✓		
Chan et al. (2007)	✓		✓	✓		✓
Dickinson et al. (2001)		✓	✓	✓		
Elfes et al. (2005)			✓	✓	✓	
Ghasemzadeh et al. (1999)		✓	✓	✓		
Gustaffson&Salo (2005)	✓	✓		✓		✓
Lee et al. (2001)	✓	✓		✓		
Luenberger (1998)		✓	✓	✓		
Norkin et al. (1998)	✓		✓	✓	✓	
Sallie (2002)		✓	✓	✓		
Utturwar et al. (2002)		✓	✓	✓		
PROPOSED APPROACH	✓	✓	✓	✓	✓	✓

Figure 3: Summary of the existing literature on project portfolio optimization and contributions of the proposed methodology

cost constraints. Goldner & Borener (2006) describe a quantitative framework to evaluate the performance of research portfolios, where the developed tool is used to evaluate and explore independent investments strategies, but no numerical optimization techniques are described. In addition to these models, most strategic planners and project portfolio managers rely on tools based on expert opinions, such as Analytical Hierarchy Process and Quality Function Deployment, in planning the funding of technology development (Thompson, 2006). Similar systematic evaluation methods are also proposed by Sallie (2002) and Utturwar *et al.* (2002), where the authors propose bilevel approaches in selecting technologies to invest. The latter study also contains an optimization procedure based on a Genetic Algorithm implementation. Clearly, these tools are also very limited in their ability to fully quantify the complicated return and investment structure inherent in project portfolios. Hence, it is essential that advanced decision tools to determine optimal project portfolios are developed.

As shown in Figure 3, our proposed methodology is able to capture all the important aspects required from a project portfolio optimization tool, as defined by Reyck *et al.* (2005), while all other existing methodologies fail to account for two or more of the complexities inherent in the project portfolio optimization problem.

CHAPTER III

STOCHASTIC NETWORK CAPACITY PLANNING

The need for the development of comprehensive stochastic network capacity planning models was discussed in Section 2.1.1. As noted, we study this problem in the context of the airport terminal capacity planning problem, since the ideas and models developed for this specific case in this chapter are applicable to the general class of network capacity problems. This generalization is also discussed in Section 3.5. None of the existing models described for network planning in Sections 1.2 and 2.1.1 address the network capacity problem in a truly holistic fashion, in large part because of the difficulty of modeling flow in a complex network structure with transient demand patterns. Furthermore, expandability is never accounted for.

In this study, we consider the airport terminal capacity planning problem, and assume that the level of service at airport terminals is measured by the total time a passenger spends in the system. This is consistent with the criteria used in most terminal design applications, where capacity is measured in terms of the processing times of passengers at different service stations (Ashford, 1988). Only those processes required for arrivals or departures are considered in total time calculations, which also include walking times. To remedy the shortcomings in existing studies, we develop time functions to approximate maximum delay in passageways and processing stations according to the patterns of peak demand. These functions for different peak demand conditions are discussed in Appendix A, and their derivations are also described in Solak *et al.* (2006). Deterministic versions of these approximating functions form convex objective functions. Using these functions, which are also valid for queuing networks and other passenger flow networks, optimal capacities corresponding to highest possible levels of service can be calculated using a stochastic programming model based on a multicommodity flow network representation of the whole airport terminal.

3.1 The Multistage Stochastic Programming Model for Airport Terminal Capacity Planning

For analysis purposes, we consider an airport terminal as a network, in which passengers with different origin-destination pairs move between nodes following predetermined demand patterns. Let $G(\mathcal{V}, \mathcal{A})$ denote this directed network, where $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ is its set of nodes and $\mathcal{A} = \{a_1, a_2, ..., a_m\}$ is its set of arcs. Each node $v_i \in \mathcal{V}$ represents either a physical location in the terminal or the arrival and departure events at a service station, such as the ticket counters or the security checkpoints. Let $\mathcal{A} = \mathcal{A}_w \cup \mathcal{A}_p$ such that $\mathcal{A}_w$ is the set of arcs between location nodes and $\mathcal{A}_p$ is the set of arcs connecting the service arrival and departure nodes. If $\mathcal{K}$ represents the set of passenger types, we assume two subsets of $\mathcal{K}$ such that $\mathcal{K} = \mathcal{K}_d \cup \mathcal{K}_p$, where $\mathcal{K}_d$ contains types of passengers that do not go through a process station to reach their destination, and $\mathcal{K}_p$ contains those that need to go through a process station. Furthermore, $\tilde{\mathcal{K}}_l$ is a subset of $\mathcal{K}_p$, which contains all passenger types that can be processed at process station $l \in \mathcal{A}_p$.

For each passenger type $k \in \mathcal{K}$, a set of nodes $\mathcal{O}^k$ contains all nodes that passenger type k can originate from and a singleton set $\mathcal{D}^k = \{v^k\}$ represents the destination, while d_i^k denotes the peak arrival rate of passengers of type k into a node $i \in \mathcal{O}^k$. Passengers of type $k \in \mathcal{K}_d$ are defined such that they have a unique origin and destination pair, so $\mathcal{O}^k$ is also a singleton set for $k \in \mathcal{K}_d$. We also define $\mathcal{R}$ as the set of process completion nodes in the network. Furthermore, $\mathcal{R}^k$ is the set of process completion nodes that passengers of type k can visit as the last process node before arriving at their destination.

In addition, let u_l denote the service capacity of a process represented by arc

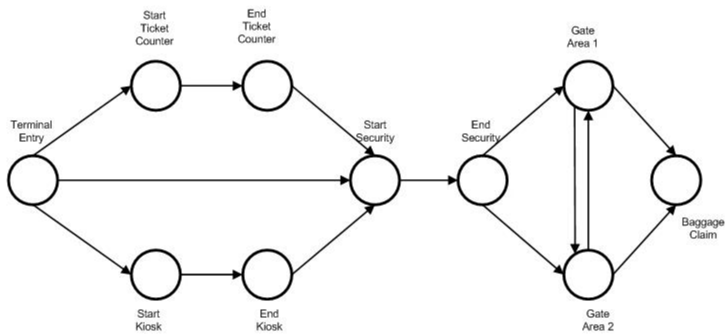

Figure 4: A simplified network representation of an airport terminal

$l \in \mathcal{A}_p$. Similarly, let w_l be the width of a passageway represented by arc $l \in \mathcal{A}_w$. Furthermore, f_l represents the peak flow rate on arc $l \in \mathcal{A}$, while x_l^k is the flow of passenger type k on arc l. Note that in problem formulation (11)-(27), the index l for each variable is replaced by (i, j), as each arc is denoted by the two nodes that it spans. For each arc $l \in \mathcal{A}$, let t_l correspond to the maximum time spent by any passenger on that arc, which, due to congestion, varies with the amount of flow on the arc according to results obtained in Appendix A. A simplified sample network representation of an airport terminal is shown in Figure 4. In this sample network, five passenger types can be considered. $\mathcal{K}_p$ contains two types with Gate Area1 and Gate Area2 as destinations, while $\mathcal{K}_d$ contains four passenger types with unique origin destination pairs of Gate Area1-Gate Area2, Gate Area2-Gate Area1, Gate Area1-Baggage Claim and Gate Area2-Baggage Claim.

The described network is similar to a multicommodity flow network, in which different types of passengers correspond to different commodities. Several objective functions can be considered for this flow model. An objective could be to find a routing for passengers through the network in such a manner that the maximum total time a passenger spends in the system for the worst case scenario is minimized for all routes. Another similar objective could be the minimization of maximum delay on each passageway and processing station. Consistent with the system equilibrium concept of Wardrop (1952), we assume that during peak demand periods, passenger flow is distributed optimally among alternate routes within the airport terminal. In the proposed model, minimization of maximum delay on each arc is chosen as the objective, and a weight factor corresponding to the arc flow rate f_l is introduced in the objective function of the model for each arc travel time function to approximate this behavior. These delay functions depend on the capacity levels, which are maximized in the optimization model given a budget B.

Expandability and the decisions on when to expand play an important role in the determination of the optimal capacity levels for a terminal building. These expansions can be realized by building a separate terminal building or by expanding the existing one to accommodate increased demand. In any case, a fixed cost β_l and a variable cost α_l will be incurred for each unit of added capacity ϵ_l on the component of the terminal represented by arc l. Assuming several planning epochs $i, i = 0, 1, 2, ..., T$ and deterministic demand forecasts, a multiperiod decision model based on the network structure described above can be formulated as the airport terminal capacity planning problem. However, such a deterministic model would not consider the variation in demand forecasts. This is a critical shortcoming as the randomness associated with demand forecasts may play a significant role in the cost-effectiveness of an expansion policy. These factors can be accounted for by considering the described model in a stochastic setting. The deterministic airport terminal capacity planning problem is NP-hard, as it can easily be shown that it contains the integer knapsack problem as a special case. Thus, the stochastic version of the airport terminal capacity planning problem is also NP-hard.

We propose a multistage stochastic integer programming model with nonlinear costs for the capacity planning problem at airport terminals. We assume that decisions on initial design capacities u_l^o for service stations and w_l^o for passageways with associated unit costs α_l^o are made while the specific scenario to occur is unknown. Expansion decisions at future planning periods are made after the realization of demand, providing recourse options.

Suppose that demand levels $d = \{d^k, k = 1, 2, ..., K\}$ between consecutive planning periods occur at one of multiple levels, i.e. low, medium and high, for each scenario. Hence, a scenario tree $\mathcal{T}$, reflecting possible realizations of demand levels over the planning periods can be constructed. Each node n of the tree corresponds to a state at some planning epoch $i = 0, 1, 2, ..., T$. The probability of being in state n is given as p_n, and we let the subscript n refer to the values of all other parameters at state n. Furthermore, we let $\mathcal{P}(n)$ represent a path from the root node 0 in the scenario tree to node n, and also $\mathcal{N}$ denote the set of non-leaf nodes. If v_{nl} is a boolean variable denoting whether an expansion on arc l is realized at the planning epoch corresponding to node n, and ϵ_{nl} is the amount of expansion, then the following stochastic program can be used to obtain the optimal capacity expansion policy under stochastic demand:

Airport Terminal Capacity Planning Problem (ATCAP):

$$\text{minimize} \sum_{n\in\mathcal{T}}\sum_{l\in\mathcal{A}_p} p_n f_{nl} t^{\rho}_{nl}(\cdot) + \sum_{n\in\mathcal{T}}\sum_{l\in\mathcal{A}_w} p_n f_{nl} t^{\omega}_{nl}(\cdot) \tag{11}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - \sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = 0 \qquad \forall n,k,\forall i\notin\{\mathcal{O}^k\cup\mathcal{D}^k\cup\mathcal{R}\} \tag{12}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} = d^k_{in} \qquad \forall n,k,\forall i\in\mathcal{O}^k \tag{13}$$

$$\sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = \sum_{i'\in\mathcal{O}^k} d^k_{i'n} \qquad \forall n,\forall k\in\mathcal{K}_d,\forall i\in\mathcal{D}^k \tag{14}$$

$$\sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = \sum_{i'\in\mathcal{R}^k}\sum_{j'\in\mathcal{V}:(i',j')\in\mathcal{A}} x^k_{n,i'j'} \qquad \forall n,\forall k\in\mathcal{K}_p,\forall i\in\mathcal{D}^k \tag{15}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - u_{n,i'i}\frac{\sum_{j\in\mathcal{O}^k} d^k_{jn}}{\sum_{k'\in\tilde{\mathcal{K}}_{i'i}}\sum_{j\in\mathcal{O}^{k'}} d^{k'}_{jn}} = 0 \qquad \forall n,\forall(i',i)\in\mathcal{A}_p,\forall k\in\tilde{\mathcal{K}}_{i'i} \tag{16}$$

$$u^o_l + \sum_{m\in\mathcal{P}(n),m\neq n} \epsilon_{ml} = u_{nl} \qquad \forall n,\forall l\in\mathcal{A}_p \tag{17}$$

$$w_l^o + \sum_{m \in \mathcal{P}(n), m \neq n} \epsilon_{ml} = w_{nl} \qquad \forall n, \forall l \in \mathcal{A}_w \tag{18}$$

$$\epsilon_{nl} - M_{nl} v_{nl} \leq 0 \qquad \forall n \in \mathcal{N}, n \neq 0, \forall l \tag{19}$$

$$\sum_{l \in \mathcal{A}} (\alpha_{nl} \epsilon_{nl} + \beta_{nl} v_{nl}) \leq B_n \qquad \forall n \in \mathcal{N}, n \neq 0 \tag{20}$$

$$\sum_{l \in \mathcal{A}_w} \alpha_l^o u_l^o + \sum_{l \in \mathcal{A}_p} \alpha_l^o w_l^o \leq B^o \tag{21}$$

$$\sum_{k \in \mathcal{K}} x_{nl}^k = f_{nl} \qquad \forall n, l \tag{22}$$

$$u_{nl} - f_{nl} \leq 0 \qquad \forall n, \forall l \in \mathcal{A}_p \tag{23}$$

$$(1 - c_{nl}) f_{nl} - u_{nl} \leq 0 \qquad \forall n, \forall l \in \mathcal{A}_p \tag{24}$$

$$Q(x_{nl}^k, u_{nl}, f_{nl}, \epsilon_{nl}) \leq 0 \qquad \forall n, k, l \tag{25}$$

$$x_{nl}^k, u_{nl}, f_{nl}, \epsilon_{nl}, u_l^o, w_l^o \geq 0 \qquad \forall n, k, l \tag{26}$$

$$v_{nl} \in \{0, 1\} \qquad \forall n \in \mathcal{N}, n \neq 0, \forall l \tag{27}$$

where $t_{nl}^{\rho}(\cdot)$ and $t_{nl}^{\omega}(\cdot)$ in (11) represent the time function associated with each process and passageway arc in the network, respectively. These functions are discussed in detail in Appendix A. Constraints (12)-(16) are node balance constraints, where (15) and (16) capture the transient behavior at process arcs during peak load periods, assuming that the departure rate form a process station is equal to the service rate of that station. Hence, f_l in the time functions of downstream arcs is determined by a proportion of the departure rate from the preceding process arcs. This proportion is defined according to the ratio of demand levels for each passenger type. In addition, (17) and (18) ensure that the total available capacity is equal to the sum of expansions made up to the current planning epoch. Constraints (19) limit the amount of expansion to M_{nl} and ensure that no expansion is made when $v_{nl} = 0$. Constraints (23) and (24) ensure that the capacity of a process station lies between the average and maximum flow rates into that station, which is a required assumption for the validity of the delay approximations used in (11). The constants c_{nl} can be estimated as

$c_{nl} = 1 - (\sum_{k\in\tilde{\mathcal{K}}_l}\sum_{i\in\mathcal{O}^k}\bar{d}^k_{in})/(\sum_{k\in\tilde{\mathcal{K}}_l}\sum_{i\in\mathcal{O}^k}d^k_{in})$, which follows from the assumption that $(\sum_{k\in\tilde{\mathcal{K}}_l}\sum_{i\in\mathcal{O}^k}\bar{d}^k_{in})/(\sum_{k\in\tilde{\mathcal{K}}_l}\sum_{i\in\mathcal{O}^k}d^k_{in}) = \bar{f}_{nl}/f_{nl}$. Constraints (20) and (21) are the budget constraints, where initial budget B^o in (21) is defined such that fixed costs are deducted from it. Finally, (25) refers to a vector of additional constraints imposed on the flows and capacities. These additional constraints may include minimum flow and capacity requirements or those that require simultaneous expansions in different areas of the terminal. Moreover, the objective function is convex for deterministic approximations.

Inputs to the above model are the peak inflow rates d^k_{in} for each level of demand realization, the cost terms α_{nl} and β_{nl}, and the arc time function expressions $t^{\rho}_{nl}(\cdot)$ and $t^{\omega}_{nl}(\cdot)$, which are discussed in Appendix A. Depending on the shape of the peak demand curve, the model can be implemented with any of the -deterministic or stochastic- processing delay approximations in Appendix A, while the half-ellipsoid approximation must be used in downstream process arcs. $ATCAP$ is a multistage stochastic integer program with linear constraints and a nonlinear objective function, independent of the use of deterministic or stochastic approximations. Since it has been observed that stochastic approximations do not provide significant improvements over deterministic ones for most instances, and that the highly nonlinear terms in the stochastic approximations would make the problem more difficult to solve when these estimates are used in the objective function, we utilize the deterministic approximations for computational studies.

3.2 An Efficient Solution Procedure for $ATCAP$

As discussed in Section 1.1.1, there are no practical general purpose algorithms for multistage stochastic integer programming problems. Although the deterministic equivalent of a stochastic integer problem can be solved by branch and bound methods, for most problem formulations the multistage structure leads to a large number of integer variables, which makes the problem difficult to solve. On the other hand, for $ATCAP$, the number of stages are limited, since each planning stage covers 4-5 years and passenger demand forecasts exist usually for 15-20 years into the future. Thus, the number of discrete variables is not so large as to prevent the solution of the deterministic equivalent of the proposed stochastic model in reasonable time. Nonetheless, efficiency of any implemented solution procedure is important, especially for general network implementations, since the capacity expansion decisions in some applications may be more frequent. In the following section, we propose a branch and bound algorithm, which is significantly efficient when compared to standard branch and bound procedures used by general purpose mixed integer nonlinear programming (MINLP) solvers. Our branch and bound algorithm relies on the implementation of an effective upper bounding heuristic at each node of the branch and bound tree.

A lower bound for $ATCAP$ can be obtained by solving the nonlinear programming (NLP) relaxation of the problem obtained by relaxing the binary capacity expansion decision variables. Since all the constraints in the model are linear, an optimal solution to this relaxed problem can be obtained in a relatively easy fashion. This lower bound can be used to obtain tight upper bounds during the branch and bound algorithm.

The solution of the NLP relaxation provides initial capacities as well as flow and capacity levels in future time periods. Although a rounding procedure can be implemented to obtain a feasible integer solution, the quality of this solution is likely to be poor. A tighter upper bound can be obtained through better heuristics. Given the solution to the NLP relaxation of $ATCAP$, we propose a heuristic based on solving a relaxed multiple choice knapsack problem. In our proposed heuristic, it is assumed that the flow levels and initial capacities are fixed according to the relaxed MINLP solution, and a feasible integer solution is obtained by determining an expansion policy that aims to maximize capacity at each node of the scenario tree, while remaining feasible according to other constraints that involve capacity levels. We let $\mathcal{S}_t$ and

$\mathcal{S}_{t+1}^n$ represent the set of nodes in scenario tree $\mathcal{T}$ that correspond to time stage t, and set of child nodes of node n, respectively. Furthermore, we let P_n represent the parent node of node n and G be a user defined scalar. Then, the following algorithm provides a feasible integer solution for $ATCAP$:

Algorithm 1. **procedure** ATCAP UPPER BOUND

Given a scenario tree $\mathcal{T}$, feasible flow $\mathbf{F}$, arc capacities $\mathbf{U}$ and arc widths $\mathbf{W}$ for relaxed $ATCAP$

for $t = 1$ to T **do**

for each $n \in \mathcal{S}_t$ **do**

for each $l \in \mathcal{A}$ **do**

Step 1. Set $\underline{\epsilon}_{nl}$ and $\bar{\epsilon}_{nl}$ to be the minimum and maximum possible expansion levels

if $l \in \mathcal{A}_p$ **then**

$\underline{\epsilon}_{nl} = \max\{0, \max_{n' \in \mathcal{S}_{t+1}^n} \{(1 - c_{nl})f_{n'l} - u_{nl}\}\}$

$\bar{\epsilon}_{nl} = \min\{M_{nl}, \min_{n' \in \mathcal{S}_{t+1}^n} \{f_{n'l} - u_{nl}\}\}$

if $\bar{\epsilon}_{nl} < \underline{\epsilon}_{nl}$ **then**

Set $t = t - 1, n^o = n, n = P_n$

Repeat Step 1 by replacing $\underline{\epsilon}_{nl} = \max\{0, \max_{n' \in \mathcal{S}_{t+1}^n} \{f_{n'l}/2 - (u_{nl} + \underline{\epsilon}_{n^o l} - \bar{\epsilon}_{n^o l}\}\}$

end if

else

if $l \in \mathcal{A}_w$ **then**

$\underline{\epsilon}_{nl} = 0$

$\bar{\epsilon}_{nl} = M_{nl} - w_{nl}$

end if

end if

for $g = 0$ to G **do**

$\epsilon_{nl}^{g} = \frac{(\overline{\epsilon}_{nl} - \underline{\epsilon}_{nl})g}{G} + \underline{\epsilon}_{nl}$

end for

end for

Step 2. Solve the LP relaxation of the following optimization problem:

$$\text{minimize } z = \delta_n \qquad \text{s.t.} \tag{28}$$

$$\sum_{l \in \mathcal{A}} \sum_{g : \epsilon_{nl}^{g} \neq 0} (\alpha_{nl}\epsilon_{nl}^{g} + \beta_{nl}) y_{nl}^{g} + \delta_n = B_n \tag{29}$$

$$\sum_{g=0}^{G} y_{nl}^{g} = 1 \tag{30}$$

$$y_{nl}^{g} \in \{0, 1\} \tag{31}$$

for each $l \in \mathcal{A}$ **do**

for each g such that $y_{nl}^{g*} \neq 0$ **do**

if $y_{nl}^{g*} = 1$ **then**

$\epsilon_{nl} = \epsilon_{nl}^{g}$

else

if $0 < y_{nl}^{g*} < 1$ **then**

$\epsilon_{nl} = \max_{g} \{\epsilon_{nl}^{g} : \sum_{g' : 0 < y_{nl}^{g'*} < 1} (\alpha_{nl}\epsilon_{nl}^{g'} + \beta_{nl}) y_{nl}^{g'*} \geq \alpha_{nl}\epsilon_{nl}^{g} + \beta_{nl}\}$

end if

end if

end for

for each $n' \in \mathcal{S}_{t+1}^{n}$ **do**

$u_{n'l} = u_{nl} + \epsilon_{nl}$ if $l \in \mathcal{A}_p$

$w_{n'l} = w_{nl} + \epsilon_{nl}$ if $l \in \mathcal{A}_w$

end for

end for

end for

end for

end procedure

The procedure above results with a feasible solution for $ATCAP$, which can be used to obtain an upper bound for the optimal objective function value. Assuming that flow levels at each time stage and initial capacities are fixed, Step 1 determines the minimum and maximum expansion levels such that the resulting capacities are feasible at each node of the scenario tree. Since any expansion will be between these bounds, this interval is divided into discrete values which are candidate expansion levels. The number of these discrete values can be determined by considering the computational burden and the level of accuracy desired in the heuristic. In Step 2, a modified relaxed version of the multiple choice knapsack problem is solved which ensures that maximum capacity expansion is achieved given the available budget. The integer variables in the optimal solution of this subproblem indicate the level of expansion on an arc for the considered node. The fractional decision variables in the solution are considered together, and the portion of budget used for these variables in the optimization problem is reallocated so that the resulting expansion policy is feasible. Capacities for the child nodes of the current node are determined by adding the expansion level on each arc to the capacity available at this node. This process is performed at all nodes in the scenario tree. If the maximum expansion level possible at a node is not sufficient to ensure feasibility at a child node, then the procedure backtracks to the parent node and resolves the expansion problem after updating the minimum expansion level at the parent node accordingly. If necessary, the process is iterated so that feasibility is always maintained.

3.3 Computational Results for $ATCAP$

Computational studies were conducted using the simplified network representation of an airport terminal in Figure 4, as well as the larger network in Figure 5, which is based on the configuration of the South Terminal at Hartsfield-Jackson Atlanta

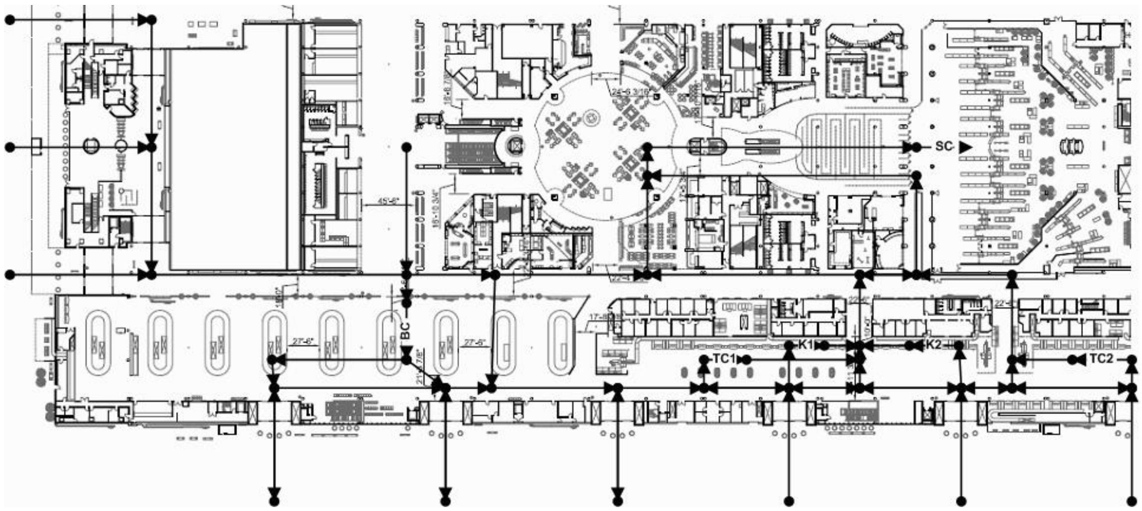

Figure 5: The network model based on the configuration of the South Terminal at Hartsfield-Jackson Atlanta International Airport

International Airport. Despite several simplifications of actual passenger flow, this larger network contains 59 passageway arcs and 6 processing arcs. In addition, 9 terminal entry points are assumed for departing passengers with a single destination node representing the completion of security screening. For arriving passengers, a single node represents the origin, while three destinations corresponding to three different terminal exit points are assumed.

In the first test model, only unidirectional flows were assumed between arcs. However, bidirectional flow was integrated into the larger model. An arrival rate curve similar to Figure 16 was assumed to be available for each customer type, and lengths of passageways were assumed to be fixed constants. The demand curves in the initial processing stations were assumed to be of triangular shape, while for the downstream processing stations a half-elliptical peak was assumed. The process delay times were estimated using the deterministic approximations in Appendix A. Hypothetical values were assumed for other input parameters, and multistage models of up to five stages were studied. Implementation of a standard branch and bound procedure as well as the improved method with the upper bounding heuristic was performed using the GAMS/SBB MINLP solver. Computations were performed on a PC with an Intel

Table 1: Performance of the upper bounding heuristic for *ATCAP*

$\|E\|$	$\|\mathcal{T}\|$	Standard B&B			B&B with Heuristic		
		Nodes	CPUs	Gap(%)	Nodes	CPUs	Gap(%)
14	4	2	0.05	-	0	0.05	-
14	13	36	3.75	-	2	0.29	-
14	40	5538	3600	8.8	63	24.83	-
14	121	2017	3600	14.4	83	639.4	-
65	4	5	0.09	-	0	0.12	-
65	13	71	10.1	-	6	3.14	-
65	40	428	1885	-	20	496.6	-
65	121	672	3600	0.8	49	2020	-

Pentium 4 1.4 GHz processor and 512MB of internal memory. A relative tolerance of 0.0001 was used, while a time limit of 1 hour was imposed on the computations. The improvements in the solution times when the upper bounding heuristic is used are shown in Table 1. In this table, first column represents the number of edges in the test problem networks, while the second column is the number of nodes in the scenario tree. Standard branch and bound implementation did not produce an optimal solution within the one hour time limit for four and five stage problems on the small network, as well as the five stage problem on the larger network. Although the running time of the heuristic increases with increasing problem size and complexity, in all instances the improved branch and bound procedure performs significantly better than the standard solution approach.

3.4 Validation of Results

The developed capacity model includes approximations of congestion effects in a pedestrian flow network due to flow levels. The validity of the proposed model is dependent on the accuracy of these approximations, which are discussed in Appendix A. We perform the validation analysis of the developed methodology by using data gathered at Hartsfield-Jackson Atlanta International Airport as a part of a peak period flow analysis study, as well as simulation results.

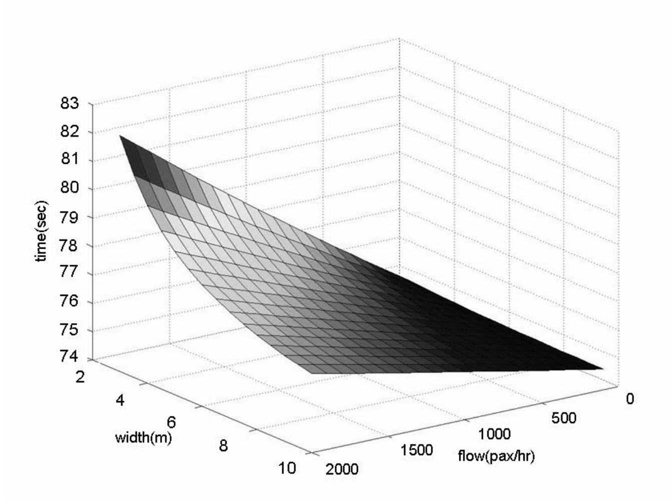

Figure 6: Walking times as a function of flow and passageway width, as calculated from relation (108) for a passageway of length L

We first consider the passageway approximations. A surface plot of maximum walking times as a function of flow and passageway width according to the developed model is shown in Figure 6. This walking time approximation has been tested and validated on a simulation model based on the walking speed relation (109) for a single passageway. Maximum walking times obtained through the simulation of different flow rates and the corresponding calculated values are displayed in Figure 7. The results suggest that as flow rate increases, the approximation (108) starts to overestimate the delay. However, the estimation is accurate for flow levels up to 4000 passengers per hour, and even for higher flow levels, the absolute error is not very significant.

For service station approximations, comparisons are shown in Figures 8 and 9 of the deterministic and stochastic approximation results for triangular and parabolic peaks respectively, with those obtained through a simulation study with Poisson arrivals and exponential processing times for a single service station. As also shown numerically in Table 2, deterministic approximations are accurate for all flow-capacity ratios and appear only to include some slight underestimation. The first column in

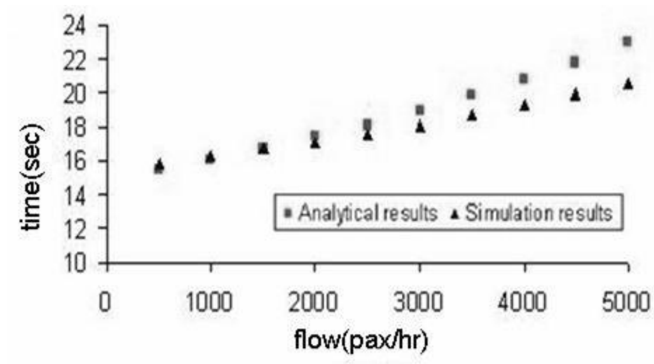

Figure 7: Comparison of passenger walking times calculated according to relation (108) with simulation results for a passageway of length L and width w

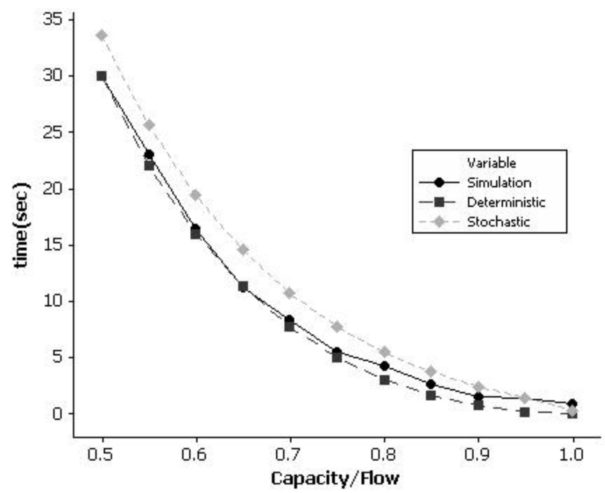

Figure 8: Comparison of deterministic and stochastic delay approximations with simulation results for a processing station with a triangular peak

this table represents the ratio of capacity versus demand rate, and the next three columns show the simulation results along with delays according to the deterministic and stochastic approximations developed for the triangular peak case. The last three columns provide the same information for the parabolic peak case. It is observed that stochastic approximations do not provide any improvement when compared with the deterministic estimations. For a single process, they tend to overestimate the maximum delay, and thus it can be concluded that deterministic approximations are valid and accurate for modeling queuing systems with a single service station.

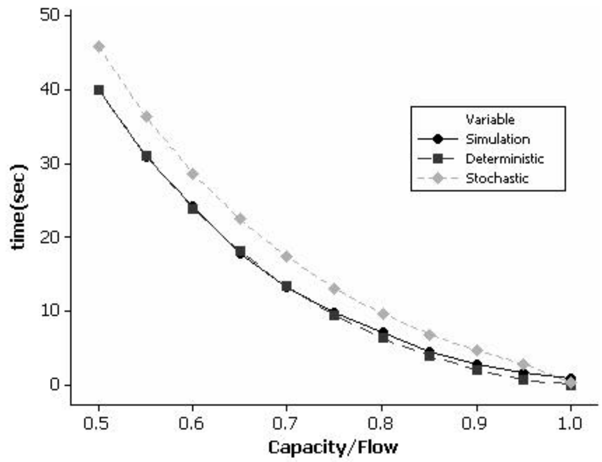

Figure 9: Comparison of deterministic and stochastic delay approximations with simulation results for a processing station with a parabolic peak

Table 2: Comparison of analytical and simulation results for triangular and parabolic peaks for a single service station

Capacity/Flow	Triangular Peak Delay(min)			Parabolic Peak Delay(min)		
	Simul.	Det. Approx.	Stoc. Approx.	Simul.	Det. Approx.	Stoc. Approx.
0.50	30.10	30.00	33.63	40.00	40.00	45.75
0.55	23.10	22.09	25.65	30.90	31.05	36.28
0.60	16.40	16.00	19.41	24.20	23.85	28.62
0.65	11.20	11.31	14.52	17.80	18.02	22.38
0.70	8.40	7.71	10.68	13.20	13.28	17.27
0.75	5.49	5.00	7.71	9.75	9.43	13.07
0.80	4.27	3.00	5.42	7.17	6.32	9.63
0.85	2.60	1.59	3.68	4.47	3.87	6.83
0.90	1.53	0.67	2.38	2.77	1.99	4.58
0.95	1.35	0.16	1.37	1.61	0.67	2.79
1.00	0.82	0.00	0.27	0.91	0.00	0.27

A more realistic setting for a validation study for the developed models is a network structure. The argument studied in Section A.3 was tested by comparing results from a simulation study with the approximations for each shape studied. The simplified network representation of an airport terminal shown in Figure 4 was simulated. In this network structure, two check-in stations are followed by the security checkpoints. Maximum delay at the security checkpoints according to simulation results have been recorded and compared with the results from delay approximations for each shape. Comparisons of the triangular and half-ellipsoid approximations with simulation results are shown in Figure 10. As seen in this figure, as well as numerically

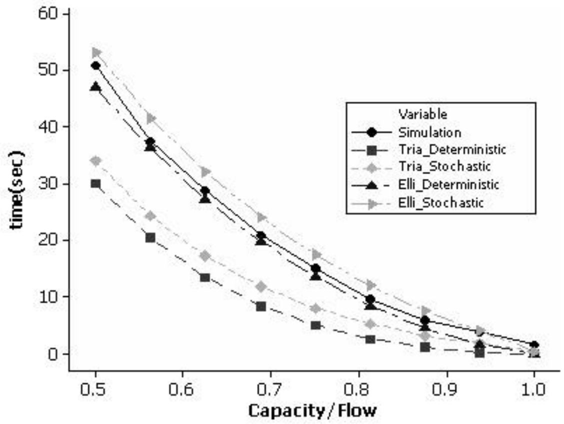

Figure 10: For downstream processes, the peaks are best approximated by a half-elliptical shape as observed from the comparison of triangular and elliptical approximations with the simulation results on a network

Table 3: Comparison of analytical and simulation results for the simplified airport terminal network structure (all times are in minutes)

Cap./Flow	Simul.	Triangular Peak Delay		Parabolic Peak Delay		Half-elliptical Peak Delay	
		Det.	Stoc.	Det.	Stoc.	Det.	Stoc.
0.500	51.10	30.00	34.07	40.00	46.44	47.10	53.26
0.563	37.5	20.42	24.37	29.10	34.82	36.35	41.78
0.625	29.00	13.50	17.22	20.78	25.90	27.36	32.21
0.688	21.00	8.52	11.93	14.37	18.95	19.85	24.20
0.750	15.10	5.00	8.04	9.42	13.51	13.60	17.50
0.813	9.64	2.60	5.22	5.65	9.27	8.48	11.98
0.875	5.85	1.07	3.21	2.85	5.98	4.45	7.50
0.938	3.92	0.25	1.77	0.94	3.48	1.52	4.03
1.000	1.66	0.00	0.32	0.00	0.32	0.00	0.32

in Table 3, both the deterministic and stochastic approximations for the triangular and parabolic peak assumptions underestimate the actual maximum delay. While stochastic version of the half-elliptical peak approximation overestimates the delay, it is observed that the deterministic half-elliptical approximation is fairly accurate, as hypothesized. Thus, arrivals at downstream processes need to be studied according to a half-elliptical peak assumption. However, in all cases, it is possible to analyze the functions individually, possibly through observations or simulation studies, and determine the best approximating shape.

In addition to simulation analyses, comparisons with observed statistics at Hartsfield Jackson Atlanta International Airport were made for validation purposes. Despite the lack of detailed data, information was obtained from peak week survey results and a time study conducted in recent years at the airport.

For passageway delay approximations, results of a concourse circulation and level of service analysis are discussed for the six concourses at Hartsfield-Jackson Atlanta International Airport in HPC (2001). In addition, Solak (2001) describes the results of a peak period time study for two of these concourses, providing walking time observations. The maximum walking times in two different 656-feet long passageways with effective widths of 12.5 feet were recorded as 184.2 seconds and 156.0 seconds under the peak flow rates of 5898 passengers/hour and 2802 passengers/hour, respectively. The corresponding approximations according to (108) for these two cases are 190.7 seconds and 166.4 seconds, which are very close to the actual observations. On the other hand, actual data for processing stations is also very limited and only available through HPC (2005). According to this study, a maximum delay of 31 minutes was observed at security checkpoints during a triangular peak demand level of 7242 passengers/hour, where the average processing capacity of the security checkpoints were calculated as 3690 passengers/hour. For this setting, the triangular peak delay approximation (118) returns an estimate of 28.3 minutes, thus confirming the closeness of the approximation.

3.5 Generalization to Stochastic Network Capacity Planning

As noted earlier, airport terminal capacity planning is a special case of the general stochastic network capacity planning problem. The multistage stochastic programming model (11)-(27) and the developed efficient solution procedure is directly applicable to this general class of problems. The only difference in a general implementation is the delay expression for transfers in the corresponding network, since approximation (108) is developed for pedestrian movements in a terminal environment. On the other hand, a large class of network design problems either contain only processing stations or are applications where delays due to flow congestion can be ignored. Hence, the developed approach is a comprehensive methodology that captures both the stochasticity in flow related parameters in a network and accounts for congestion in a system where flow patterns are transient. In addition to capacity allocation, the model is also applicable to scenario analyses for different network configurations and robust network design. Furthermore, the developed multistage model can easily be reduced to a two-stage problem, if such an analysis is deemed more appropriate as discussed in Section 1.2.

CHAPTER IV

PROJECT PORTFOLIO OPTIMIZATION

The project portfolio management problem can formally be defined as follows. Assume a set $\mathcal{N}$ of projects or technologies with annual performance levels $Z_i \in \mathbb{R}^+$, implementation times $\Delta_i \in \mathbb{R}^+$, required investment levels $\theta_i \in \mathbb{R}^+$, annual fixed activity costs $f_i \in \mathbb{R}^+$ and a set of depending technologies $\mathcal{D}_i \subset \mathcal{N}$, for each $i \in \mathcal{N}$. Although only two-way dependencies between technologies are used in this study, the proposed models can be extended to handle multi-way dependencies in a similar fashion. We let $Z_{ij} \in \mathbb{R}$ be the joint annual performance level for technology $i \in \mathcal{N}$ and $j \in \mathcal{D}_i$, and define it as a function of Z_i and Z_j. Furthermore, a sequence of investment planning periods $t = 1, 2, ..., T$ with available resource levels, i.e. budgets $B_t \in \mathbb{R}^+$, are assumed. For presentation purposes, the models in the paper are described for a single resource application, however extension to multiple resources is trivial. The objective is to determine an investment schedule such that some function of the total discounted return over an infinite time horizon is maximized while total investment in a given period t does not exceed B_t. In typical applications, the decision maker is interested in the investment schedule for the current period only, which should take into account future realizations of the parameters. Hence, a realistic assumption is that the problem will be solved each planning period to determine the best investment policy for that period, considering the past and future investments.

In practice, almost all of the above parameters may contain a certain level of uncertainty. However, in most applications, the level of variance is significant only in two of the parameters, namely the returns Z_i and required investment levels θ_i . Note that Z_{ij} is defined as a function of Z_i and Z_j. Hence, for modeling purposes, we approximate all other parameters with their expected values, and assume that joint and marginal probability distributions of the returns and required investment levels for the technologies are known or well estimated. Once a mathematical model is developed that accounts for the stochasticity in these two parameters, uncertainty in other parameters can be captured through what-if analyses.

The following complexity analysis shows that even the simplest instances of the project portfolio management problem fall into the category of NP-hard optimization problems.

Proposition 1. *Project portfolio management is NP-hard.*

Proof. We first show that the deterministic version of the problem is NP-hard. The proof of NP-hardness is by restriction to the bin packing problem. Consider an instance of the project portfolio management problem in which $B_t = B$, $\theta_i + f_i \leq B$, $\Delta_i = 0$, and $\mathcal{D}_i = \emptyset$ for all $i \in \mathcal{N}$,$t = 1, 2, ..., |\mathcal{N}|$, . Let $\mathcal{S}^*$ be the optimal schedule for this instance and let t^* be the latest investment period in $\mathcal{S}^*$. It is easily seen that $\mathcal{S}^*$ is optimal if and only if the optimal solution for an instance of the bin packing problem with bin capacities B and item sizes $\theta_i + f_i$ is t^*.

It follows that stochastic version of the project portfolio management problem is also NP-hard. □

Before we discuss modeling and solution approaches to the more difficult stochastic project portfolio optimization problem, we first consider the deterministic version and develop a mixed integer linear programming formulation for the problem. The deterministic model helps in gaining insights for the solution of the problem, and assessing heuristic policies that can be used to allocate resources to the projects.

4.1 A Mixed Integer Programming Model for the Deterministic Project Portfolio Optimization Problem

In addition to the parameters described above, we let r be the discount factor throughout the planning period, and $\mathcal{D}$ be the set of technology projects that have a dependency relationship with another project, i.e. $\mathcal{D} = \{i | i \in \mathcal{N}, \mathcal{D}_i \neq \emptyset\}$. Without loss of generality, we assume that only two-way dependencies exist between projects. Furthermore, we define the following decision variables for the problem:

x_{it} :amount of investment in project i in period t, $t = 1, 2, ..., T$

z_{it} :discounted return of project i in period t, $t = 1, 2, ..., T + \Delta_i$

z_{ijt} :discounted joint return of projects i and j in period t, $t = 1, 2, ..., T + \max\{\Delta_i, \Delta_j\}$

τ_{it} :remaining required investment to complete the development of project i as of the end of period t, $t = 1, 2, ..., T$

α_{it} :1, if technology project i is started on or before period t, $t = 1, 2, ..., T$; 0, otherwise

β_{it} :1, if development and deployment of technology i are completed on or before period t, $t = 1, 2, ..., T$; 0, otherwise

δ_{ijt} :1, if development and deployment of dependent technologies i and j are completed on or before period t, $t = 1, 2, ..., T + \max\{\Delta_i, \Delta_j\}$; 0, otherwise

We assume that investment decisions are made at the beginning of a planning period, and the returns are discounted and accrued at the end of a period. Then, the following mixed integer programming formulation can be developed for the deterministic project portfolio optimization problem:

Deterministic Project Portfolio Optimization Problem(D-PROPT):

$$\text{maximize} \sum_{i\in\mathcal{N}}\sum_{t\le T+\Delta_i} z_{it} + \sum_{i\in\mathcal{D}}\sum_{\substack{j\in\mathcal{D}_i\\ j>i}}\sum_{t\le T+\max\{\Delta_i,\Delta_j\}} z_{ijt} \tag{32}$$

$$\alpha_{it} - \beta_{i,t+\Delta_i} \ge 0 \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{33}$$

$$\sum_{t'\le t} x_{it'} - (\theta_i + tf_i)\alpha_{it} \le 0 \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{34}$$

$$\sum_{i\in\mathcal{N}} x_{it} \le B_t \qquad \forall t\le T \tag{35}$$

$$x_{it} - B_t(\alpha_{it} - \beta_{i,t+\Delta_i-1}) \le 0 \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{36}$$

$$z_{i,t+\Delta_i} - \frac{Z_i}{r}(1+r)^{-(t+\Delta_i)} (\beta_{i,t+\Delta_i} - \beta_{i,t+\Delta_i-1}) \le 0 \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{37}$$

$$z_{i,t+\Delta_i} - \frac{Z_i}{r}(1+r)^{-(t+\Delta_i)}(1-\beta_{j,t+\Delta_i}) + \sum_{t'>t+\Delta_i}\frac{Z_i}{r}(1+r)^{-t'}(\beta_{jt'} - \beta_{j,t'-1}) \le 0 \qquad \forall i\in D, \forall j\in D_i, \forall t\le T \tag{38}$$

$$z_{ij,t+\max\{\Delta_i,\Delta_j\}} - \frac{Z_{ij}}{r}(1+r)^{-(t+\max\{\Delta_i,\Delta_j\})} (\delta_{ij,t+\max\{\Delta_i,\Delta_j\}} - \delta_{ij,t+\max\{\Delta_i,\Delta_j\}-1}) \le 0 \qquad \forall i\in D, \forall j\in D_i, j>i, \forall t\le T \tag{39}$$

$$\beta_{i,t+\max\{\Delta_i,\Delta_j\}} + \beta_{j,t+\max\{\Delta_i,\Delta_j\}} - \delta_{ij,t+\max\{\Delta_i,\Delta_j\}} \le 1 \qquad \forall i\in D, \forall j\in D_i, j>i, \forall t\le T \tag{40}$$

$$\beta_{i,t+\max\{\Delta_i,\Delta_j\}} + \beta_{j,t+\max\{\Delta_i,\Delta_j\}} - 2\delta_{ij,t+\max\{\Delta_i,\Delta_j\}} \ge 0 \qquad \forall i\in D, \forall j\in D_i, j>i, \forall t\le T \tag{41}$$

$$\tau_{it} - \tau_{i,t-1} + x_{it} - f_i(\alpha_{it} - \beta_{i,t+\Delta_i-1}) = 0 \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{42}$$

$$x_{it} - f_i(\alpha_{it} - \beta_{i,t+\Delta_i-1}) \ge 0 \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{43}$$

$$\tau_{it} + \theta_i\beta_{i,t+\Delta_i} \le \theta_i \qquad \forall i\in\mathcal{N}, \forall t\le T \tag{44}$$

$$x_{it}, \tau_{it}, z_{it}, z_{ijt} \ge 0 \qquad \forall i\in\mathcal{N}, \forall j\in D_i, j>i, \forall t \tag{45}$$

$$\alpha_{it}, \beta_{it}, \delta_{ijt} \in \{0,1\} \qquad \forall i\in\mathcal{N}, \forall j\in D_i, j>i, \forall t \tag{46}$$

Constraint set (33) implies that technology development project i must be started at least Δ_i periods before its implementation is complete. Constraints (34) ensure that no investment is made on a technology development project before the project is started. Furthermore, (36) require that an investment on a technology development project can be made only if it is active, while (35) are the budget constraints. Constraints (37)-(39) define individual and joint returns for the projects, while (40)-(41) ensure that joint return from two dependent projects is realized when the implementation of both projects are complete. (42) calculates the required remaining investment for a technology development project in a given period, and (43) implies that the investment on a project can not be less than the fixed costs incurred when the project is active. Finally, constraints (44) ensure that a technology development project is complete only if the required remaining investment is 0.

4.1.1 Computational Results for $D - PROPT$

Since Monte Carlo simulation based solution procedures are likely approaches for the stochastic portfolio optimization problem, it is important that the mixed integer linear program developed for $D - PROPT$ is solved fast on commercially available solvers. To investigate the solution times for problem instances and to demonstrate the validity of the developed model, several test problems with up to ten projects and ten investment periods were generated and solved using ILOG CPLEX Version 10.0 on a PC with an Intel Pentium 4 1.4 GHz processor and 512MB of internal memory. A relative tolerance of 0.0001 was used for solution accuracy. With default CPLEX settings, the average solution time for the ten project problems was observed to be approximately 7 seconds, which is promising for use of Monte Carlo methods. However, special procedures will need to be developed for real-world instances of the $D - PROPT$, since these instances may usually contain more technology projects. The data for a sample instance with ten technologies, and the corresponding solution

Table 4: Data for a sample instance of $D - PROPT$ with ten technologies

Attributes / Projects	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Fixed activity cost (mil.$)	0.2	0.1	0.3	0.2	0.2	0.05	0.1	0.2	0.05	0.3
Required investment (mil.$)	3	4	5	4	2	2	6	1	3	2
Implementation time (yrs)	5	2	3	3	2	3	1	4	2	2
Annual return (mil.$)	1.5	3.5	1.5	3.5	0	2	7	2	1.5	4.5
Dep./Joint return (mil.$)	-	C/4	B/4	E/6	D/6	-	J/8	-	-	G/8

Table 5: Solution of the sample instance of $D - PROPT$ with ten technologies

Projects / Years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12...15**
A									0.9	2.5	I	I
B			0.9	0.8	2.6	I	I					
C												
D	0.7	1.8	2.1	I	I	I						
E				2.2	I	I						
F					0.4	1.7	I	I	I			
G						1.3	3	2	I			
H		1.2	I	I	I	I						
I								1	2.1	I	I	
J	2.3	I	I									
Budget	3	3	3	3	3	3	3	3	3	3		
Total Invested	3	3	3	3	3	3	3	3	3	2.5		

for the instance are given in Table 4 and Table 5, respectively. The entry "I" in Table 5 denotes the implementation or deployment of a technology in the corresponding period after its development. Since the actual technology portfolio optimization problem is highly stochastic, solution of the deterministic model only serves as a tool to view the structure of the decision making process and the optimal solutions. For instance, it is important to note that the optimal investment schedule for the sample instance does not suggest investing first in the technology with the highest return.

4.1.2 Deterministic Formulation with Resource Usage Minimization and Implementation Deadlines

A second practical problem in technology project portfolio management is the determination of required resource levels that will ensure that technologies are developed and deployed by certain deadlines. In this version of the problem, two objectives exist, which are the minimization of required resource levels and maximization of the total expected return subject to the deadline constraints. We first assume that the decision maker will be first asked to decide on the relative importance of return maximization versus resource usage minimization by assigning weights to each objective, which are denoted by $\sum_{t\leq T}\lambda_t$ and $1-\sum_{t\leq T}\lambda_t$ in the formulation below. Then, the decision maker will make an importance ordering of the periods for resource allocation by selecting appropriate values for λ_t. It is also assumed that an upper bound $\bar{B}_t$ exists for each decision variable B_t. If Y_i is the latest time period that technology i needs to be implemented by, then these properties can be captured by changing the objective function and adding new constraints as follows:

Deterministic Project Portfolio Optimization Problem with Resource Minimization (D-PROPT-RM):

$$\text{maximize } \big(1-\sum_{t\leq T}\lambda_t\big)\Big(\sum_{i\in\mathcal{N}}\sum_{t\leq T} z_{it} + \sum_{i\in\mathcal{D}}\sum_{\substack{j\in\mathcal{D}_i\\ j>i}}\sum_{t\leq T+\max\{\Delta_i,\Delta_j\}} z_{ijt}\Big) - \sum_{t\leq T}\lambda_t B_t \qquad (47)$$

$$(33)-(46)$$

$$\beta_{i,Y_i}=1 \qquad \forall i\in\mathcal{N} \qquad (48)$$

$$B_t\leq \bar{B}_t \qquad \forall t\leq T \qquad (49)$$

$$B_t\geq 0 \qquad \forall t\leq T \qquad (50)$$

The implementation deadlines for technologies are enforced by constraints (48). These constraints automatically enforce joint implementation deadlines, since a joint deadline is simply a combination of two individual deadlines. Note that time value of resources can be accounted for by incorporating a present value term with a discount factor in the resource component of the objective function. This modified model can easily be solved similar to the deterministic project portfolio model discussed above.

4.2 Stochastic Project Portfolio Optimization Problem

Given the uncertainty in the problem parameters of the project portfolio optimization problem, it is natural to assume that the decision maker would be interested in maximizing the expected value -or a function of the expected value- of total return. For presentation purposes, we assume a risk-neutral objective function throughout the rest of this paper. However, several other objectives that capture the risk attitude of the decision maker can be modeled and solved using the methods described in this study. Given any such objective, the project portfolio management problem can be expressed as:

$$\max_{\mathbf{x}\in\mathcal{X}}\{g(\mathbf{x}) = \mathbb{E}[G(\mathbf{x},\xi)]\} \tag{51}$$

where $\mathbf{x}$ and ξ represent the vectors of decision variables and uncertain parameters (θ_i, Z_i), respectively. In addition, $\mathcal{X} \subset \mathbb{R}^n$ is the set of feasible solutions and $G(\mathbf{x},\xi)$ is the total return function. Optimization problem (51) is difficult to solve, since exact evaluation of the expected value function in the objective is not possible.

A natural temptation to solve (51) may involve replacing the uncertain parameters by their expected values, and then solving the resulting so-called mean value problem, which is

$$\max_{\mathbf{x}\in\mathcal{X}}\{G(\mathbf{x},\bar{\xi})\} \tag{52}$$

where $\bar{\xi} = \mathbb{E}[\xi]$ is the expectation of the random vector ξ. If $\bar{x}$ represents the optimal solution to (52), and x^* is the true optimal solution to the stochastic optimization problem (51), then clearly

$$\mathbb{E}[G(\bar{x},\xi)] \leq \mathbb{E}[G(x^*,\xi)] \tag{53}$$

The difference $\mathbb{E}[G(x^*,\xi)] - \mathbb{E}[G(\bar{x},\xi)]$ measures how close the mean value solution is to the true solution, and is usually called the expected value of the stochastic solution (Birge & Louveaux, 1997). However, the mean value problem usually does not reflect the decision process in a stochastic optimization problem correctly.

The decision process in the project portfolio management problem consists of recourse actions, by which the portfolio can be rebalanced at each period. Hence, an appropriate approach is to formulate problem (51) as a recourse problem, in which

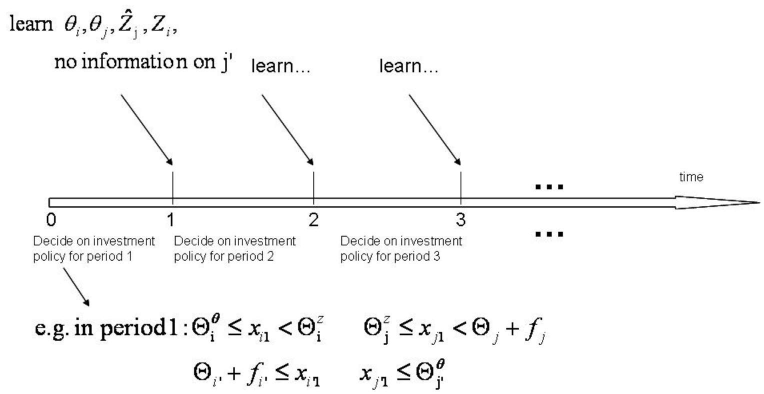

Figure 11: Decision process for the technology portfolio management problem, where realization of uncertainty is based on decisions made

recourse actions can be taken after uncertainty is disclosed over the investment periods. In the following sections, we study two recourse models for the project portfolio management problem, and describe solution procedures for the two formulations.

4.2.1 The Multistage Stochastic Programming Model for Project Portfolio Optimization

The decision making process in the project portfolio management problem consists of a multistage and multi-period structure, in which the goal is to determine an optimal allocation of the resources for the current planning period. However, the realization of uncertain parameters and the possibility of recourse actions in future periods must be accounted for in any optimal investment policy. Hence, resource allocations for the current period should position the decision maker in the best possible position against the uncertainties that will be realized in the future. The decision process for the project portfolio management problem can be described as follows, which is also represented in Figure 11, where examples of different investment levels leading to different information availability for projects i, i', j and j' are shown.

The resource requirement θ_i for each project i is known with certainty at the end of period t_θ^i, in which total investment in the project exceeds a threshold level Θ_i^θ, i.e. $t_\theta^i = \min_t\{t | \sum_{t' \le t} x_{it'} \ge \Theta_i^\theta\}$, where x_{it} represents the investment for project

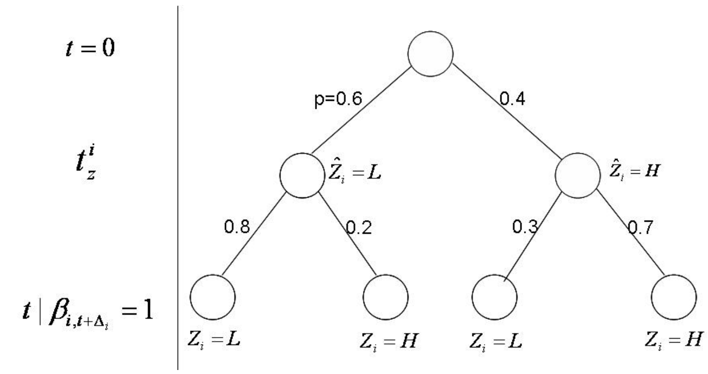

Figure 12: Tree showing gradual resolution of uncertainty in two phases

i in period t. Similarly, we assume that the uncertainty in the return of a project is revealed gradually over its development based on certain threshold levels. This process is modeled by assuming that an initial performance assessment $\hat{Z}_i$ will be available at the end of period $t_z^i = \min_t\{t \mid \sum_{t' \leq t} x_{it'} \geq \Theta_i^z\}$ upon investing an amount of Θ_i^z in the technology. As a result of this assessment, probabilities of different performance levels are updated. This assumption enables the modeling of the option of terminating a project if the initial assessment suggests that the probability of a high return is low for the technology. Gradual resolution of uncertainty can be explained further as follows. Assume that Z_i can be realized at one of two levels: L, H with pre-development probabilities p_L and p_H, respectively. After investing an amount Θ_i^z in this technology, an estimate $\hat{Z}_i$ is made, which can be seen as an intermediate realization of the uncertain parameter. If all uncertainty is resolved when technology development is over, then the probabilities for the actual realizations of the possible outcomes will be dependent on the intermediate realizations. This investment dependent probability distribution is described in Figure 12, where probabilities of possible Z_i values are updated according to the estimates $\hat{Z}_i$ which become available after investing Θ_i^z units of resources. If the development phase is continued, return Z_i will be known with certainty once all of the required resources are invested in technology i. Multiple phases of gradual uncertainty resolution can be modeled by adding more layers to the described process, however this will add more complexity to the stochastic problem.

The described process can be modeled as a multistage stochastic program, in which the uncertainty is in required investment levels, updated return estimates and final return levels. However, a complexity in this model is that the model contains endogenous uncertainty, i.e. realizations of the uncertain parameters are dependent on the investment decisions in the current and future periods. Classical stochastic programming models assume that all stochastic processes in a given model are exogenous, which implies that the times of realizations of the uncertain parameters are not controlled by the decision maker, and the underlying scenario tree structure is known. However, this is not the case for the project portfolio management problem, and so it is in the class of multistage stochastic programs with endogenous uncertainty, where, in addition, uncertainty in some parameters is gradually resolved according to the values of continuous decision variables. These problems are generally more difficult to formulate and solve than classical stochastic programming models, and there is very limited literature on such problems, which we discuss in Section 2.2.

As in many other stochastic programs, it is reasonable to assume for the project portfolio optimization problem that the random vector ξ has finite support or has a discrete distribution with K possible realizations, i.e. scenarios, $\xi^k := (\theta_i^k, \hat{Z}_i^k, Z_i^k)$, $k = 1, \ldots, K$ with corresponding probabilities p_k. Then, it becomes possible to express problem (51) as one large mathematical program.

Before describing the mathematical model, we introduce some further notation. We first set $\bar{\Delta} = \max_i\{\Delta_i\}$, $\bar{\Delta}_{ij} = \max\{\Delta_i, \Delta_j\}$. We also let $Y_{kk'}$ and $H_{kk'}$ be the set of projects with different realizations of resource requirements and intermediate return estimates in scenarios k, k', respectively, i.e. $Y_{kk'} = \{i | \theta_i^k \neq \theta_i^{k'}\}$ and $H_{kk'} = \{i | \hat{Z}_i^k \neq \hat{Z}_i^{k'}\}$. Furthermore, we define the following decision variables for the problem, where the superscript k, which indicates that the variables are defined for each scenario, is omitted for clarity .

y_{it} :1, if $t > t^i_\theta$, $t = 2, ..., T$; 0, otherwise

h_{it} :1, if $t > t^i_z$, $t = 2, ..., T$; 0, otherwise

γ_{it} :1, if project i is terminated prematurely in or before period t, $t = 2, ..., T$

0, otherwise

This leads to the following multistage stochastic integer programming formulation: *Multistage Project Portfolio Management Problem(MPPM):*

$$\max \sum_{k=1}^{K} p_k \sum_{i\in\mathcal{N}} \Big[\sum_{t\le T-1} \beta^k_{i,t+\Delta_i} Z^k_i (1+r)^{-(t+\Delta_i)} + \beta^k_{i,T+\Delta_i} Z^k_i \big[\frac{(1+r)^{-(T+\bar{\Delta})}}{r} + \sum_{l=0}^{\bar{\Delta}-\Delta_i-1} (1+r)^{-(T+\Delta_i+l)} \big] + \sum_{\substack{j\in\mathcal{D}_i\\ j>i}} \Big[\sum_{t\le T-1} \delta^k_{ij,t+\bar{\Delta}_{ij}} \widetilde{Z}^k_{ij} (1+r)^{-(t+\bar{\Delta}_{ij})} + \delta^k_{ij,T+\bar{\Delta}_{ij}} \widetilde{Z}^k_{ij} \big[\frac{(1+r)^{-(T+\bar{\Delta})}}{r} + \sum_{l=0}^{\bar{\Delta}-\bar{\Delta}_{ij}-1} (1+r)^{-(T+\bar{\Delta}_{ij}+l)} \big] \Big] \Big] \quad (54)$$

$$\alpha^k_{it} - \beta^k_{i,t+\Delta_i} \ge 0 \qquad \forall i\in\mathcal{N}, \forall t\le T, \forall k \quad (55)$$

$$\sum_{t'\le t} x^k_{it'} - \big(\max\{(\theta^k_i + tf_i), \max_{t'\le t}\{B_{t'}\}\big)\, \alpha^k_{it} \le 0 \qquad \forall i\in\mathcal{N}, \forall t\le T, \forall k \quad (56)$$

$$\sum_{i\in\mathcal{N}} x^k_{it} \le B_t \qquad \forall t\le T, \forall k \quad (57)$$

$$x^k_{it} - B_t(\alpha^k_{it} - \beta^k_{i,t+\Delta_i-1} - \gamma^k_{it}) \le 0 \qquad \forall i\in\mathcal{N}, \forall t\le T, \forall k \quad (58)$$

$$\beta^k_{i,t+\bar{\Delta}_{ij}} + \beta^k_{j,t+\bar{\Delta}_{ij}} - \delta^k_{ij,t+\bar{\Delta}_{ij}} \le 1 \qquad \forall i\in D, \forall j\in D_i, j>i, \forall t\le T, \forall k \quad (59)$$

$$\beta^k_{i,t+\bar{\Delta}_{ij}} + \beta^k_{j,t+\bar{\Delta}_{ij}} - 2\delta^k_{ij,t+\bar{\Delta}_{ij}} \ge 0 \qquad \forall i\in D, \forall j\in D_i, j>i, \forall t\le T, \forall k \quad (60)$$

$$\tau^k_{it} - \tau^k_{i,t-1} + x^k_{it} - f_i(\alpha^k_{it} - \beta^k_{i,t+\Delta_i-1} - \gamma^k_{it}) \ge 0 \qquad \forall i\in\mathcal{N}, \forall t\le T, \forall k \quad (61)$$

$$x^k_{it} - f_i(\alpha^k_{it} - \beta^k_{i,t+\Delta_i-1} - \gamma^k_{it}) \ge 0 \qquad \forall i\in\mathcal{N}, \forall t\le T, \forall k \quad (62)$$

$$\tau^k_{it} + \theta^k_i \beta^k_{i,t+\Delta_i} \le \theta^k_i \qquad \forall i\in\mathcal{N}, \forall t\le T, \forall k \quad (63)$$

$$\sum_{t'<t} x^k_{it'} - \Theta^\theta_{ik} y^k_{it} \geq 0 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, t \neq 1, \forall k} \quad (64)$$

$$\sum_{t'<t} x^k_{it'} - \left(\min\{\textstyle\sum_{t'<t} B_{t'}, (\theta^k_i + (t-1)f_i)\} - \Theta^\theta_{ik}\right) y^k_{it} \leq \Theta^\theta_{ik} \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, t \neq 1, \forall k} \quad (65)$$

$$\sum_{t'<t} x^k_{it'} - \Theta^z_{ik} h^k_{it} \geq 0 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, t \neq 1, \forall k} \quad (66)$$

$$\sum_{t'<t} x^k_{it'} - \left(\min\{\textstyle\sum_{t'<t} B_{t'}, (\theta^k_i + (t-1)f_i)\} - \Theta^z_{ik}\right) h^k_{it} \leq \Theta^z_{ik} \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, t \neq 1, \forall k} \quad (67)$$

$$\beta^k_{i,t+\Delta_i} + \gamma^k_{it} \leq 1 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, t \neq 1, \forall k} \quad (68)$$

$$\alpha^k_{it} - \gamma^k_{i,t+1} \geq 0 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, \forall k} \quad (69)$$

$$x^k_{i1} - \sum_{k'=1}^{K} p_{k'} x^{k'}_{i1} = 0 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall k} \quad (70)$$

$$x^k_{it} - x^{k'}_{it} + B_t \left[\sum_{j \in Y_{kk'}} (y^k_{jt} + y^{k'}_{jt}) + \sum_{j \in H_{kk'}} (h^k_{jt} + h^{k'}_{jt}) \right] \geq 0 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t \leq T, t \neq 1, \forall k, k'} \quad (71)$$

$$x^k_{it}, \tau^k_{it}, \delta^k_{ijt} \geq 0 \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall j \in D_i, j > i, \forall t, \forall k} \quad (72)$$

$$\alpha^k_{it}, \beta^k_{it}, \gamma^k_{it}, h^k_{it}, y^k_{it} \in \{0,1\} \qquad {\scriptstyle \forall i \in \mathcal{N}, \forall t, \forall k} \quad (73)$$

The objective function (54) in the above formulation assumes risk neutrality, and represents the expected total discounted return of the project portfolio. The total return is expressed as a function of the individual and joint returns depending on the implementation status of a technology. Joint return terms $\widetilde{Z}_{ij}$ are defined such that they represent the difference between the actual joint return contribution Z_{ij} and the sum of two individual returns. In other words, if two technologies are both implemented by period t, then the joint return contribution for that period is calculated as $Z_{ij} = Z_i + Z_j + \widetilde{Z}_{ij}$, where $\widetilde{Z}_{ij}$ can be positive or negative.

Constraint set (55) implies that project i must be started at least Δ_i periods before it is implemented. Constraints (56) ensure that a positive investment must be made in order to start a technology development project. Furthermore, (58) requires that an investment on a technology development project can be made only if it is active, while (57) represents the resource limitations. Constraints (59)-(60) ensure that joint return from two dependent technologies is realized when the implementation of both technologies are complete. Constraints (61) calculate the required remaining investment for a technology development project in a given period, and (62) implies that the investment on a technology development project can not be less than the fixed cost incurred when the project is active. Constraints (63) ensure that a technology development project is complete only if the required remaining investment is 0. Furthermore, constraints (64)-(65) and (66)-(67) define indicator variables y_{it}^k and h_{it}^k, respectively. Constrains (68) state that a technology development project is either terminated successfully or unsuccessfully, while (69) ensures that a project is started before it is terminated.

In addition to the above, constraint set (70) represents the first stage nonanticipativity requirements, by ensuring that the decisions for the current period are the same for all scenarios. Notice that the nonanticipativity in other first stage variables are automatically satisfied if all first stage investment levels are the same. Since it is assumed that the uncertain variables are realized after certain levels of investment are made, a similar nonanticipativity structure must also be enforced between scenarios that share the same information history in later periods. In classical stochastic programming, nonanticipativity can explicitly be stated similar to (70), due to the exogenous nature of uncertainty in these problems. Since the uncertainty is endogenous in the project portfolio management problem, the nonanticipativity is conditional on the investment level decisions in each planning period. Constraints (71) capture this dependency by ensuring that a given pair of scenarios will be distinguished when one or more of the uncertain variables that distinguish them are revealed. The time of realization of the uncertainty is determined by the binary variables y_{it}^k and h_{it}^k. Notice that (71) are defined as inequalities for each possible pair of technologies so that if no distinguishing parameters are known, then the investment levels in the two technologies have to be equal. In addition, assuming independence of the corresponding probability distributions, any two scenarios that differ only in the realization of the final return values will have the same investment policy, since all investment decisions are made before these realizations. Hence, the return levels do not play a role in the nonanticipativity requirements. Representation of endogenous nonanticipativity in this compact way is distinct and more efficient than the existing models in the literature, since it enables the use of scenario decomposition methods as well as some other solution approaches proposed for classical multistage stochastic integer programming problems.

4.2.2 An Efficient Solution Procedure for $MPPM$

One way of introducing endogeneity into stochastic programming models is by defining a boolean decision vector d in the first stage, the value of which determines the time of realization for the random parameters. Notice that if d is fixed, the scenario tree for the multistage stochastic program will be known for this specific instance. Thus the optimization problem reduces to finding d that will provide the maximum objective function value when the corresponding multistage stochastic programming problem is solved, i.e.

$$\max_{d} \max_{X} v = \mathbf{E}^{d}[F(X_{\omega}, \Xi_{\omega})] \tag{74}$$

s.t.

$$g_j(X_{\omega}, \Xi_{\omega}) \leq 0 \qquad j = 1, 2, ..., m; \quad \omega \in \Omega \tag{75}$$

$$X \in \mathbf{X}^{d} \tag{76}$$

Solving this problem involves enumerating all decision vectors d and solving the corresponding multistage stochastic programming problems. However, this would be practically impossible, since each multistage program by itself is difficult to solve due to the large number of scenarios. This leads to the selection of sampling based methods as a preferred practical solution procedure for problems of this type.

Monte Carlo sampling methods have been studied extensively for stochastic optimization problems. Existing methods include the infinitesimal perturbation analysis technique (Ho & Cao, 1991), stochastic approximation (Kushner & Clark, 1978), statistical L-shaped method (Infanger, 1994) and the stochastic decomposition method (Higle & Sen, 1996). Another common procedure is sample average approximation (SAA), which separates the optimization phase from the sampling phase, thus making it easy to implement in large complex problems (Rubinstein & Shapiro, 1990).

The sample average approximation method is a Monte Carlo sampling technique that approximates a stochastic program by a smaller problem based on a random sample from the set of possible scenarios. Let $\xi^1, ..., \xi^N$ be an i.i.d. random sample of N realizations of the random vector ξ. Then the SAA problem for (51) is:

$$\max_{\mathbf{x} \in \mathcal{X}} \{\hat{g}_N(\mathbf{x}) = \frac{1}{N} \sum_{l=1}^{N} G(\mathbf{x}, \xi^l)\} \tag{77}$$

If v^* and $\hat{v}_N$ represent the optimal values of the "true" and SAA problems respectively, it is well known that $\hat{v}_N$ is a valid upper statistical bound for v^* . Furthermore, Shapiro (2003) shows that for multistage stochastic programming problems $\hat{v}_N$ converges to v^* with probability 1 as $N \to \infty$, although no result is available on the rate of convergence. Hence, the choice of large values of N will lead to better approximations of the true objective function. However, since the computational complexity of the SAA problem increases exponentially with the value of N, it is more efficient to select a smaller sample size N, and solve several SAA problems with i.i.d. samples.

Let M represent the number of SAA problems solved, and let $\hat{v}_N^m$ and $\hat{\mathbf{x}}_N^m$, $m = 1, \ldots, M$, denote the optimal objective value and solution of the mth replication, respectively. Since only the first stage investment decisions have practical importance for the project portfolio management problem, we assume that $\hat{\mathbf{x}}_N^m$ represents these first stage decisions. Once a feasible solution $\hat{\mathbf{x}}_N^m \in \mathcal{X}$ is obtained by solving the SAA problem, the objective value $g(\hat{\mathbf{x}}_N^m)$ can be approximated by the unbiased estimator

$$\hat{g}_{N'}(\hat{\mathbf{x}}_N^m) = \frac{1}{N'} \sum_{l=1}^{N'} G(\hat{\mathbf{x}}_N^m, \xi^l) \tag{78}$$

where N' is typically larger than N, since the computational effort required to estimate the objective value for a given solution is generally less than that required to solve the SAA problem. On the other hand, this phase may also be difficult for multistage problems, since it requires solving a multistage problem with endogenous uncertainty where only the first stage decisions are known. Hence, any solution procedure must especially be efficient in calculating $\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)$. One would also want to estimate the quality of the solution $\hat{\mathbf{x}}_N^m$. This can be done by computing an estimate of the optimality gap $v^* - g(\hat{\mathbf{x}}_N^m)$, where $g(\hat{\mathbf{x}}_N^m)$ can be estimated by (78), and v^* can be approximated by

$$\bar{v}_N^M = \frac{1}{M} \sum_{m=1}^{M} \hat{v}_N^m \tag{79}$$

The sampling procedure can be terminated once the optimality gap estimate is sufficiently small or after performing all M replications, and the best solution among the SAA solutions can be selected using an appropriate criterion. However, the variance of the optimality gap estimator is also important, and must be taken into account in determining the quality of a solution. One option, as noted by Kleywegt *et al.* (2002), is to add a multiple z_α of the estimated standard deviation of the gap estimator to the gap estimator, where $z_\alpha = \Phi^{-1}(1-\alpha)$ and $\Phi(z)$ is the cumulative distribution function of the standard normal distribution. If the sample sizes are not large, then z_α can be replaced by $t_{\alpha,\nu}$ from the t-distribution, where ν is the corresponding degrees of freedom. Then, an adjusted optimality gap estimator can be calculated by

$$\bar{v}_N^M - \hat{g}_{N'}(\hat{\mathbf{x}}_N^m) + z_\alpha \left(\hat{\sigma}^2_{\bar{v}_N^M} + \hat{\sigma}^2_{\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)} \right)^{1/2} \tag{80}$$

where $\hat{\sigma}^2_{\bar{v}_N^M}$ and $\hat{\sigma}^2_{\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)}$ are the estimates of the variances for the estimators of v^* and

$g(\hat{\mathbf{x}}_N^m)$, respectively, and are calculated as

$$\hat{\sigma}^2_{\bar{v}_N^M} = \frac{1}{(M-1)M} \sum_{m=1}^{M} (\hat{v}_N^m - \bar{v}_N^M)^2 \tag{81}$$

$$\hat{\sigma}^2_{\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)} = \frac{1}{(N'-1)N'} \sum_{l=1}^{N'} \left(G(\hat{\mathbf{x}}_N^m, \xi^l) - \hat{g}_{N'}(\hat{\mathbf{x}}_N^m) \right)^2 \tag{82}$$

Effective implementation of the above sampling procedure requires that the SAA problems can be solved efficiently for relatively large values of the sample size N. For a given set of scenarios, (54)-(73) is a difficult mixed integer programming problem and applications of standard solution methods fail to produce a solution even when N is set to values less than ten. As an efficient solution procedure for the SAA problem, we propose a Lagrangian relaxation and decomposition scheme coupled with an efficient lower bounding heuristic, which we name as the feasible dual conversion algorithm. The development of such a procedure is especially important, since for most multistage stochastic problems, even finding a feasible solution to serve as a lower bound is difficult. We show in Section 4.2.2.2 that the minimum feasible dual conversion heuristic is an effective procedure in calculating tight lower bounds for the project portfolio management problem.

4.2.2.1 A Lagrangian Relaxation and Decomposition Scheme

Model (54)-(73) is linked in scenarios through the nonanticipativity constraints (70) and (71). Let $\hat{g}_N(\beta, \delta)$ represent the objective function (54) with K and p_k replaced by N and $1/N$, respectively. Then by subjecting the nonanticipativity conditions to Lagrangian relaxation, we form the following Lagrangian

$$L(\beta, \delta, x, y, h, \lambda, \mu) = \hat{g}_N(\beta, \delta) + \sum_{l=1}^{N} \sum_{i \in \mathcal{N}} \lambda_i^l \left[\sum_{l'=1}^{N} \frac{1}{N} x_{i1}^{l'} - x_{i1}^l \right]$$
$$+ \frac{1}{N} \sum_{l=1}^{N} \sum_{l' \neq l} \sum_{i \in \mathcal{N}} \sum_{1 < t \leq T} \mu_{it}^{ll'} \left[x_{it}^l - x_{it}^{l'} + B_t \left[\sum_{j \in Y_{ll'}} \left(y_{jt}^l + y_{jt}^{l'} \right) + \sum_{j \in H_{ll'}} \left(h_{jt}^l + h_{jt}^{l'} \right) \right] \right] \tag{83}$$

where λ_i^l and $\mu_{it}^{ll'}$ are the Lagrange multipliers. Notice that the formulation of the nonanticipativity constraints (70) and the multiplication of the relaxed constraints (71) by $\frac{1}{N}$ in the above Lagrangian account for the scenario probabilities, and prevent the ill-conditioning in the Lagrangian dual as discussed by Louveaux & Schultz (2003). A major advantage of the described formulation of the nonanticipativity constraints is that when they are relaxed, the Lagrangian (83) can be decomposed by scenarios for given dual vectors λ and μ , and can be expressed as

$$L(\beta, \delta, x, y, h) = \sum_{l=1}^{N} L_l(\beta_l, \delta_l, x_l, y_l, h_l) \tag{84}$$

where

$$\begin{aligned} L_l(\beta_l, \delta_l, x_l, y_l, h_l) = \hat{g}_N^l(\beta, \delta) &+ \sum_{i \in \mathcal{N}} \left[\sum_{l'=1}^{N} \frac{\lambda_i^{l'} x_{i1}^l}{N} - x_{i1}^l \lambda_i^l \right] \\ &+ \frac{1}{N} \sum_{i \in \mathcal{N}} \sum_{1 < t \leq T} \left[x_{it}^l \sum_{l' \neq l} \left(\mu_{it}^{ll'} - \mu_{it}^{l'l} \right) + B_t \Big[\sum_{l' \neq l} \sum_{j \in Y_{ll'}} y_{jt}^l \left(\mu_{it}^{ll'} + \mu_{it}^{l'l} \right) \right. \\ &\left. + \sum_{l' \neq l} \sum_{j \in H_{ll'}} h_{jt}^l \left(\mu_{it}^{ll'} + \mu_{it}^{l'l} \right) \Big] \right] \end{aligned} \tag{85}$$

The corresponding Lagrangian dual problem for problem (54)-(73) is then

$$\min_{\lambda,\mu} \{ D(\lambda, \mu) = \max \{ \textstyle\sum_{l=1}^{N} L_l(\beta_l, \delta_l, x_l, y_l, h_l, \lambda_l, \mu_l) : (55) - (69), (72), (73), \mu_l \geq 0 \} \} \tag{86}$$

Problem (86) is a nonsmooth convex minimization problem which can be solved by subgradient optimization methods (Hiriart-Urruty & Lemarechal, 1993). At each iteration of these methods, the solution of $D(\lambda, \mu)$ is required to obtain a subgradient. Notice that $D(\lambda, \mu)$ is separable, and reduces to the solving N problems of manageable size, each of which corresponds to a single scenario. Components of the subgradient vector are then given by $\frac{\lambda_i^{l'} x_{i1}^l}{N} - x_{i1}^l \lambda_i^l$ and $x_{it}^l \sum_{l' \neq l} \left(\mu_{it}^{ll'} - \mu_{it}^{l'l} \right) + B_t \left[\sum_{l' \neq l} \sum_{j \in Y_{ll'}} y_{jt}^l \left(\mu_{it}^{ll'} + \mu_{it}^{l'l} \right) + \sum_{l' \neq l} \sum_{j \in H_{ll'}} h_{jt}^l \left(\mu_{it}^{ll'} + \mu_{it}^{l'l} \right) \right]$, where x_{i1}^l, y_{it}^l and h_{it}^l are the optimal solutions to the scenario subproblems.

For the project portfolio optimization problem, we propose a modified subgradient algorithm, in which step sizes in updating the dual variables are determined according to a weighted combination of the subgradients from previous iterations. More specifically, a new step direction at iteration j is determined by

$$\hat{\Gamma}^j = \pi_0\Gamma^j + \pi_1\Gamma^{j-1} + \pi_2\Gamma^{j-2} + \pi_3\Gamma^{j-3} \tag{87}$$

where Γ terms represent the subgradients and π terms are weights such that $\pi_0+\pi_1+\pi_2+\pi_3 = 1$. Individual π values can be selected according to an experimental analysis based on the problem considered. Updates of the multipliers are then performed using the following combined dynamic step size rule:

$$\lambda^{j+1} = \lambda^j - \max\{\frac{\phi}{j}, \frac{\kappa(\bar{L}^j - \underline{L}^j)}{||\hat{\Gamma}^j||}\}\hat{\Gamma}^j \tag{88}$$

$$\mu^{j+1} = \max\{0, \mu^j - \max\{\frac{\phi}{j}, \frac{\kappa(\bar{L}^j - \underline{L}^j)}{||\hat{\Gamma}^j||}\}\hat{\Gamma}^j\} \tag{89}$$

where ϕ and κ, $\kappa < 2$, are constants that can be modified during the algorithm. Above rule, which has been verified through computational studies, ensures that initial step sizes are large enough to prevent early convergence to a non-optimal solution. The implementation of the overall solution algorithm includes frequent lower bound calculations during the iterations of the subgradient method, and the convergence rate of the subgradient algorithm is especially important from an overall computational perspective. Hence, the stepsizes are determined as efficiently as possible to improve the convergence rate of the algorithm. Despite the large size of the dual vector for realistic instances of the problem, computational studies have shown that the convergence of the subgradient algorithm is relatively fast. Results of the tested models are discussed in Section 4.2.4.

It is well known that, due to the integrality requirements, the optimal solution of the Lagrangian dual gives an upper bound for the objective value of (54)-(73), which is at least as tight as the bound obtained from the LP relaxation of the problem.

Furthermore, any Lagrangian dual solution is an upperbound for the original problem. However, a major difficulty in solving multistage stochastic programming problems is to determine good feasible solutions for tight lower bounds. Clearly, except in rare cases, the solutions of the Lagrangian dual will not satisfy the nonanticipativity constraints.

We present a heuristic procedure that uses the Lagrangian dual solutions in subgradient iterations to search for a feasible solution to the primal problem, which provides a lower bound for the optimal objective value. Given a Lagrangian dual solution, the method looks for a primal solution with minimum deviation from the dual solution. The search, which has produced very tight bounds in the computational studies described in Section 4.2.4, is implemented using integer programming models of manageable size. To ease the computational difficulty, the procedure is implemented gradually using subsets of scenarios, which are determined by the variable values and the objective value contributions of the scenarios in the dual solution. This procedure, which can also be applied as a bounding procedure in similar stochastic programming problems, is described in detail below.

4.2.2.2 The Feasible Dual Conversion Algorithm

The objective function of the project portfolio management problem is defined by the values of the binary variables β_{it}, which represent the periods that the return realizations begin. Hence, the corresponding values in a given Lagrangian dual solution describe some infeasible investment policy in which nonanticipativity constraints are not enforced but are only penalized. Clearly, the optimal objective value of the primal problem is expected to be as close as possible or comparable to that of this infeasible policy. Although, due to the combinatorial nature of the problem, the optimal investment policy in the presence of nonanticipativity can be significantly different than the policy suggested by the given dual solution, one can obtain a "good" investment policy by converting the dual solution into a feasible solution by a minimal change in the β_{it} values in the Lagrangian dual solution. We present below an algorithm to achieve this, as well as a bound on the quality of the solution obtained through the algorithm. The feasible dual conversion algorithm performs such conversions in a systematic way that ensures the quality of the resulting solution as well as computational efficiency.

Algorithm 2 (Feasible Dual Conversion)**.** The steps of the algorithm are as follows:

Step 1. *Initialization* : Let β^j represent the vector of corresponding values in a solution to the Lagrangian dual problem (86) at iteration j of the subgradient algorithm for dual variables λ^j and μ^j. Let $\underline{\beta}^l_{it}$, $\underline{\hat{g}}_N$, $\underline{L}_l$ be the lowerbounds on β^l_{it}, $\hat{g}_N$ and L_l for scenario l. Choose a scenario subset size S. Set $\underline{\beta}^l_{it} = 0$ for all i, t, l, $\mathbb{S} = \emptyset$, $\mathbb{S}' = \emptyset$, $\mathbb{N} = \{l_1, l_2, \ldots, l_N\}$.

Step 2. *Scenario subset selection* : Rank all $s \in \mathbb{N}$ according to scenario objectives L^j_s, and form subset $\mathbb{S}$ by selecting the first S scenarios among the ranked scenarios in $\mathbb{N}$. Let $\mathbb{S}' = \mathbb{S}' \cup \mathbb{S}$ and $\mathbb{N} = \mathbb{N} \backslash \mathbb{S}$.

Step 3. *Variable fixing* : For each $s \in \mathbb{S}$, determine period t^s_o in which s becomes distinguishable from all other scenarios according to scenario solutions β^s_{it}, i.e.

$$t^s_o = \min_t \{t | \min_{s' \neq s} \{ \sum_{j \in Y_{ss'}} (\beta^s_{j,t+\Delta_j} + \beta^{s'}_{j,t+\Delta_j}) + \sum_{j \in H_{ss'}} (\beta^s_{j,t+\Delta_j} + \beta^{s'}_{j,t+\Delta_j}) \} \geq 1 \} \qquad (90)$$

For each $i \in \mathcal{N}$ such that $\beta^s_{i,t+\Delta_i} = 1$, and $t \leq t^s_o$; if $\beta^s_{i,t+\Delta_i} - \beta^s_{i,t+\Delta_i - 1} = 1$, then set $\underline{\beta}^s_{i,t+\Delta_i} = 1$.

Step 4. *Feasibility determination*: Check feasibility of (54)-(73) with the lower bounds on β^s_{it} for the scenario set $\mathbb{S}'$. If feasible, let $\dot{\beta}^s_{it}$ represent the corresponding values in this solution, and fix $\beta^s_{it} = \dot{\beta}^s_{it}$. If $\mathbb{N} \neq \emptyset$, go to Step 2.

Step 5. *Minimum dual conversion* : If (54)-(73) is infeasible, determine the minimum number of relaxations r_o required on $\underline{\beta}^s_{it} = 1$ for $s \in \mathbb{S}$ to obtain a feasible solution. Find the best possible feasible solution that can be achieved by relaxing at most r_o of the bounds $\underline{\beta}_{it}^s$. Fix $\beta_{it}^s = \dot{\beta}_{it}^s$. If $\mathbb{N} \neq \emptyset$, go to Step 2.

Step 6. *Bound calculation* : Let $\dot{\mathbf{x}}$ and $\dot{g}_N$ represent the final solution vector and objective function value. If $\dot{g}_N > \underline{\hat{g}}_N$, set $\underline{\hat{g}}_N = \dot{g}_N$. For each scenario l, calculate $\dot{L}_l(\dot{\mathbf{x}}, \lambda^{j+1}, \mu^{j+1})$. If $\dot{L}_l > \underline{L}_l^{j+1}$, set $\underline{L}_l^{j+1} = \dot{L}_l$.

After the initialization of the algorithm in Step 1 according to a Lagrangian dual solution obtained in a subgradient iteration, Step 2 identifies the scenarios with the maximum possible contribution to the total expected return. Notice that the subset selection heuristic can be replaced by any other selection procedure, such as selecting the scenarios according to groups. One can form groups of scenarios $S_1, S_2, \ldots$ such that $l, l' \in S_j$ implies $\mathbf{x}_{i1}^l = \mathbf{x}_{i1}^{l'}$. Then, for each group S_j, a representative scenario s' such that $L_{s'} = \max_{s \in S_j}\{L_s\}$ can be selected. In Step 3, projects that determine nonanticipativity relationships and that are also likely to deviate from the Lagrangian solution are identified. The β_{it} variables for these technologies are fixed so that they are completed on or before the time suggested by the ideal policy from the dual solution. Almost in all cases, this will lead to an infeasible solution, which is checked in Step 4. Then, a conversion procedure is implemented in Step 5. In this phase, first the minimum number of relaxations on the fixed β_{it} variables required to obtain a feasible solution is determined by solving an integer programming problem, which is assumed to be easily solvable for scenario subset size S. Note that such a feasible solution always exists. Another option is to minimize a weighted sum of the relaxations, where the weights are determined by the contribution of each technology into the overall objective function. Then, given this minimum requirement for feasibility, an optimization is performed to determine the best possible solution by performing at most that many relaxations on fixed β_{it} variables. Again, it is assumed that such an optimization can be performed efficiently for S scenarios. The procedure is repeated $\frac{N}{S}$ times, which results with a feasible solution for the primal problem. In Step 6, bounds on the objective values are updated to simplify the solution process in later iterations.

Indeed, in the overall implementation, a history of all such solutions are maintained, and used to determine the best possible lowerbound on scenario subproblems at each iteration. Despite the additional memory requirement, it has been observed that this significantly reduces the solution times for the scenario subproblems.

One may think that a better approach would be such that all β_{it} values in the Lagrangian dual solution are fixed in Step 3. However, this may significantly increase the computational complexity of the optimization problems solved in Step 5. Also, by minimizing the number of required relaxations, Step 6 minimizes the computational difficulty of the subsequent optimization problem, and the deviation from the dual solution is kept minimal with respect to technologies with the highest return levels. The following propositions define a bound on the quality of the solution produced by the feasible dual conversion algorithm, which translates to an upper bound on the duality gap.

Proposition 2. *Let i^s represent a project i in scenario s, and let I_β be the set containing all i^s such that $\underline{\beta_{it}^s} = 1$ for some t. For a given set $\mathbb{S}$ of scenarios, group i^s according to the order of completion in the dual scenario solutions, i.e. projects completed first in each scenario represent a group, as well as those completed second, third, etc. In case of ties, assign groups arbitrarily. Let R^n, $n \leq |\mathcal{N}|$, represent the cardinality of the largest compatibility set in group n, where projects i^s and $j^{s'}$ are defined to be in the same compatibility set if $\underline{\beta_{i,t+\Delta_i}^s} = \underline{\beta_{j,t+\Delta_j}^{s'}} = 1$, $\beta_{i,t+\Delta_i}^s = 1$ does not imply $\beta_{j,t+\Delta_j}^{s'} = 0$ or vice versa, and if they are compatible with the projects in the maximum cardinality compatibility set in group $n-1$. Then, for any application of the feasible dual conversion algorithm on $\mathbb{S}$,*

$$r_o \leq |I_\beta| - \sum_{n \leq |\mathcal{N}|} R^n$$

Proof. Clearly, an upper bound on r_o is $|I_\beta|$. Note that, to obtain feasibility, a relaxation of the lower bound on β_{it}^s or $\beta_{jt}^{s'}$ is required if i^s and $j^{s'}$ are not compatible.

Hence, required number of relaxations for each group will be minimum if β_{it}^s is set to 1 for all members of the maximum cardinality compatibility set, and the variables corresponding to the remaining projects in the group are relaxed. By the definition of compatibility, a feasible solution can always be obtained by fixing $\sum_{n\leq|\mathcal{N}|} R^n$ of the β_{it}^s variables, where $i^s \in I_\beta$, at their lowerbounds. Hence an upper bound on the number of relaxations required for feasibility is $|I_\beta| - \sum\limits_{n\leq|\mathcal{N}|} R^n$. □

The above bound on the number of relaxations is easy to calculate, since the size of the groups formed in the bound calculation procedure is in the order of S. The procedure requires the identification of the maximum cardinality compatibility set, which is equivalent to solving the NP-hard maximum clique problem on a compatibility graph. As noted, the size of the groups enable easy determination of this set. On the other hand, less tight bounds can be obtained by using bounds known for the maximum clique problem and selecting a clique arbitrarily to fix some of the variables. Proposition 3 uses the bound on r_o to develop a bound for the quality of the solutions obtained by the feasible dual conversion algorithm.

Proposition 3. *Consider a ranking of projects $i^s \in I_\beta$, i.e. $< i^s_{(1)}, i^s_{(2)}, \cdots >$ such that $z_{i^s_{(1)}} \geq z_{i^s_{(2)}} \geq \ldots$, where z_i^s is the contribution of project i to the scenario objective in the dual solution. Define $r_o^U = |I_\beta| - \sum_{n\leq|\mathcal{N}|} R^n$, and let $t^s_{o'}$ represent the period that scenario s becomes distinguishable from all other scenarios according to a modified dual solution obtained by assuming no investment is made in project i^s prior to period $t+1$, if $\underline{\beta_{it}^s} = 1$ and $i^s \in \{i^s_{(1)}, i^s_{(2)}, \ldots, i^s_{(r_o^U)}\}$ or $\underline{\beta_{it}^s} = 0$. Furthermore, assume that z_c^s represents the return in scenario s from the optimum single-scenario investment schedule over periods $t^s_{o'}, \ldots, T$ for all projects that are not completed by $t^s_{o'}$ according to the modified dual solution. For the optimum partial schedule calculations, assume that for i^s such that $\underline{\beta_{it}^s} = 0$ for all t, $\theta_i = \tau_{i,t^s_{o'}}$, if $\tau_{i,t^s_{o'}} < \theta_i$ in the modified dual solution and all x_{it}^s satisfy modified nonanticipativity for $t \leq t^s_{o'}$. If*

$F_N^*(x)$ is the optimal objective function value for the SAA problem with N scenarios, and $F_N(\dot{x})$ is the objective value of a solution generated by the feasible dual conversion algorithm,then

$$F_N^*(x) - F_N(\dot{x}) \leq \sum_{k=1}^{N/S} \sum_{s \in \mathbb{S}^k} \{-z_c^s + \sum_{i=i^s_{(1)}}^{i^s_{(r_o^U)}} z_i^s\}$$

Proof. Consider the first iteration of the feasible dual conversion algorithm, and assume that S scenarios with highest scenario objectives are selected. Notice that an upperbound for the contribution of these scenarios in the optimal solution is given by $\sum_{s=1}^{S} L_s(x, \lambda, \mu)$. Let ΔZ represent the total change in the objective value of the feasible solution for scenario s compared with the dual solution. Clearly, $\Delta Z \leq \sum_{i=i^s_{(1)}}^{i^s_{(r_o)}} z_i^s$, since a feasible solution always exists with r_o relaxations on the bounds $\underline{\beta_{it}^s} = 1$. Without loss of generality, assume that these relaxations correspond to i^s with the highest contributions to the objective function. We show that the modified dual solution described above is feasible. Suppose this solution is not feasible, which implies that the corresponding investment schedule does not satisfy the modified nonanticipativity requirements. Since the modified dual solution consists only of projects with $\underline{\beta_{it}^s} = 1$, any change in the schedule would require a relaxation in these bounds. This contradicts with the condition that a feasible solution exists with r_o relaxations on the bounds. Furthermore, any partial investment schedule for periods after $t^s_{o'}$ would not violate feasibility, since there is no nonanticipativity requirements after period $t^s_{o'}$. Hence, it is possible to improve this feasible solution by reoptimizing the allocations in each scenario s for periods after $t^s_{o'}$. This will lead to an improvement of $\sum_s z_c^s$ in the objective value, implying that $\Delta Z \leq \sum_{i=i^s_{(1)}}^{i^s_{(r_o)}} z_i^s - \sum_s z_c^s$. It follows from Proposition 2 that the bound can be expressed similarly by replacing r_o with r_o^U. Since the algorithm performs N/S iterations to obtain a feasible solution, the total difference is the sum over all iterations, and the result follows. $\square$

Calculation of the above bound requires the solution of small optimization problems for each scenario. These problems include only a subset of the projects in the portfolio, and contain periods after $t^s_{o'}$. Noting that these small problems can be solved significantly fast, the difficulty of bound calculations is only dependent on the number of scenarios considered.

Using the bounding schemes discussed, a branch and bound algorithm with branching on the nonanticipativity constraints that are not satisfied in the solution of the Lagrangian dual can be implemented to close the duality gap. In the case of the project portfolio management problem, the nonanticipativity constraints are on the continuous variables x^k_{it}. Hence a branching rule could use the average investment in the scenario solutions of the dual problem, or the most frequent occurrence of x^k_{it} values to branch on. However, the branch and bound scheme is usually computationally efficient only for very small scale problems. On the other hand, duality gaps are not significantly high for the approximate solutions produced by the feasible dual conversion algorithm for larger models as noted in Tables 7 and 8. Thus, in most instances, it will suffice to obtain approximate solutions through the feasible dual conversion algorithm, and use them as the solutions to the SAA problems. In parallel with this analysis, computational studies in Section 4.2.4 have been implemented without the branch and bound step for efficiency purposes.

4.2.2.3 Solution Algorithm Overview

The overall procedure to solve the project portfolio optimization problem is summarized below, which is also shown in Figure 13.

Algorithm 3 (Solution Algorithm for $MPPM$ and $2PPM$)**.** The general solution algorithm can be summarized as follows:

Step 1. Obtain N samples from the set of scenarios, and form the SAA problem with these scenarios.

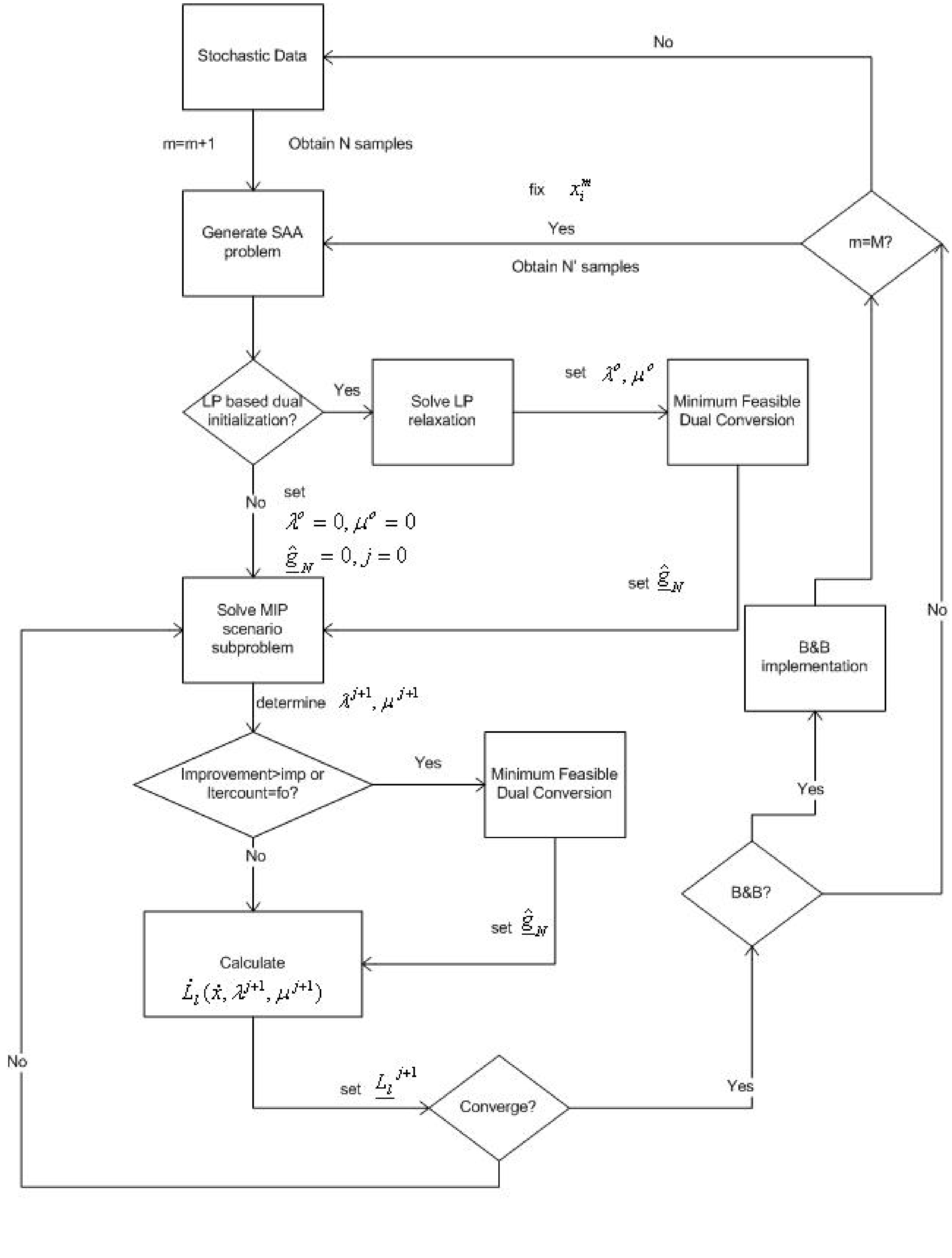

Figure 13: Solution algorithm for $MPPM$ and $2PPM$

Step 2. Perform Lagrangian relaxation on the SAA problem, decomposing the problem into individual scenario subproblems.

Step 3. Use subgradient algorithm with the proposed step size measure to obtain an upper bound for the SAA problem.

3a. If computationally feasible, solve the LP relaxation of (54)-(73), and set the corresponding dual values as the initial Lagrangian multipliers. Use a rounding heuristic to obtain an initial lowerbound on the problem, i.e if $\beta_{it}^{k} \geq 0.5$ and $\beta_{it'}^{k} \geq 0.5$ for all $t' > t$ in the LP relaxation solution, then set $\dot{\beta}_{it}^{k} = 1$, else set $\dot{\beta}_{it}^{k} = 0$. Then use the feasible dual conversion algorithm.

3b. At each iteration j of the algorithm, determine a lowerbound for the scenario subproblems by calculating $\dot{L}_l(\dot{\mathbf{x}}^l, \lambda^{j+1}, \mu^{j+1})$, and selecting the minimum.

3c. Based on an improvement threshold for the dual solution or at every f_o iterations, apply the feasible dual conversion algorithm, to obtain a lowerbound for the SAA problem, as well as for the scenario subproblems.

3d. Use the best lowerbounds for the scenario subproblems as the starting solution for the subproblems at iteration $j+1$.

4. Calculate the duality gap upon convergence of the subgradient algorithm. If the gap is less than or equal to ϵ, go to step 5. Else, if computationally feasible, use branch and bound to close the duality gap, by branching on the nonanticipativity conditions.

5. Repeat Steps 1-4 M times. Each solution is a candidate solution for the true problem.

6. For some or all of the candidate solutions, perform N' replications by fixing the values of the first stage variables according to the solution, and repeating steps $1-4$ with these fixed values to estimate the objective value of the candidate solutions.

7. Select a solution as the best solution using an appropriate criterion.

For the lower bounding procedure, both the LP relaxation based and dual solution based heuristics can be applied and the maximum objective value can be selected as the better lowerbound. Computational studies have shown that the LP relaxation based heuristic can often produce good solutions.

4.2.3 The Two-stage Stochastic Programming Model and Solution

Although the actual decision making process for the project portfolio optimization problem contains multiple stages, a natural simplification is through a two stage approach, in which it is assumed that a realization of the random variables becomes known after investment decisions are made for the current period in the first stage. If $\mathbf{x}_1$ represents the first period decision variables and $\mathbf{x}_2$ is the vector of variables for the second stage which contains the remaining $T-1$ periods, then the corresponding two stage stochastic program can be written as follows:

$$\max_{\mathbf{x}_1} \mathbb{E}[G(\mathbf{x}_1, \xi)] \tag{91}$$

$$\text{s.t. } A\mathbf{x}_1 = b, \mathbf{x}_1 \in \mathcal{X}_1 \tag{92}$$

where $G(\mathbf{x}_1, \xi)$ is the optimal value of the second stage problem

$$\max_{\mathbf{x}_2} g(\omega)^T \mathbf{x}_2 \tag{93}$$

$$\text{s.t. } T(\omega)\mathbf{x}_1 + W(\omega)\mathbf{x}_2 = h(\omega), \mathbf{x}_2 \in \mathcal{X}_2 \tag{94}$$

In the above representation, the second stage problem (93)-(94) depends on the realization ω of the random vector ξ, which determines the values of g, T, W and h.

This leads to the following two-stage stochastic integer programming formulation for the project portfolio management problem:

Two-stage Project Portfolio Management (2PPM)

$$\max\ (54) \tag{95}$$

$$(55), (57), (59), (60), (63), (70), (72) \tag{96}$$

$$\sum_{t' \le t} x_{it'}^k - \left(\max\{(\theta_i^k + tf_i), B_1\}\right) \alpha_{it}^k \le 0 \qquad \forall i \in \mathcal{N}, \forall t \le T, \forall k \tag{97}$$

$$x_{it}^k - B_t(\alpha_{it}^k - \beta_{i,t+\Delta_i-1}^k - \gamma_{i2}^k) \le 0 \qquad \forall i \in \mathcal{N}, \forall t \le T, \forall k \tag{98}$$

$$\tau_{it}^k - \tau_{i,t-1}^k + x_{it}^k - f_i(\alpha_{it}^k - \beta_{i,t+\Delta_i-1}^k - \gamma_{i2}^k) \ge 0 \qquad \forall i \in \mathcal{N}, \forall t \le T, \forall k \tag{99}$$

$$x_{it}^k - f_i(\alpha_{it}^k - \beta_{i,t+\Delta_i-1}^k - \gamma_{i2}^k) \ge 0 \qquad \forall i \in \mathcal{N}, \forall t \le T, \forall k \tag{100}$$

$$\beta_{i,t+\Delta_i}^k + \gamma_{i2}^k \le 1 \qquad \forall i \in \mathcal{N}, \forall t \le T, t \ne 1, \forall k \tag{101}$$

$$\alpha_{i1}^k - \gamma_{i2}^k \ge 0 \qquad \forall i \in \mathcal{N}, \forall t \le T, \forall k \tag{102}$$

$$\alpha_{it}^k, \beta_{it}^k, \gamma_{i2}^k \in \{0,1\} \quad \forall i \in \mathcal{N}, \forall t, \forall k \tag{103}$$

In terms of formulation, the two-stage problem differs from the multistage model only in the definition of the constraints that involve the termination variables, since early termination decisions can only be made in the second stage. Similar to the $MPPM$, if the number of scenarios K is not large, problem (96)-(103) can be solved using standard integer programming methods. However, this is not possible for realistic instances of the project portfolio management problem, since they constitute much larger problems. A difference between $MPPM$ and $2PPM$ in terms of problem size is that, the cardinality of the scenario set is smaller in $2PPM$, since gradual revelation of uncertainty is not modeled. Even then, an instance of the problem with $|\mathcal{N}| = 10$ projects, with 2 possible realizations for each of the corresponding random parameters result with $K = 2^{20}$ scenarios.

The same solution procedure described in Section 4.2.2.3 can efficiently be utilized for $2PPM$. Except that the nonanticipativity is only restricted to the first stage, so the Lagrangian is given as

$$L(\beta, \delta, x, \lambda) = \hat{g}_N(\beta, \delta) + \sum_{l=1}^{N} \left[\sum_{i \in \mathcal{N}} \left(\sum_{l'=1}^{N} \frac{\lambda_i^{l'} x_{i1}^l}{N} - x_{i1}^l \lambda_i^l \right) \right] \tag{104}$$

which can be expressed as

$$L(\beta, \delta, x) = \sum_{l=1}^{N} L_l(\beta_l, \delta_l, x_l,) \tag{105}$$

where

$$L_l(\beta_l, \delta_l, x_l) = \hat{g}^l_N(\beta, \delta) + \sum_{i \in \mathcal{N}} \left(\sum_{l'=1}^{N} \frac{\lambda_i^{l'} x_{i1}^l}{N} - x_{i1}^l \lambda_i^l \right) \tag{106}$$

The corresponding Lagrangian dual problem for problem (96)-(103) is then

$$\min_\lambda \{D(\lambda) = \max\{\textstyle\sum_{l=1}^N L_l(\beta_l, \delta_l, x_l, \lambda_l) : (96) - (103), \text{except}(70)\}\} \tag{107}$$

Computational results and the efficiency of the solution procedure for $2PPM$ are discussed in Section 4.2.4.

4.2.4 Computational Results for $MPPM$ and $2PPM$

Computational tests for the developed solution procedures were conducted on two sets of project portfolio data under different algorithmic configurations. The data sets consist of five and ten technology projects and are represented as 5T and 10T in the results tables. The stochastic data for the ten project instance is shown in Table 6. The probability distributions for the uncertain parameters, i.e. required investment levels, initial return estimates and realized return levels, were assumed to be discrete with low and high levels. Corresponding probabilities for each case are also listed in Table 6. Although the dependence of the probability distributions of return estimates and realizations are modeled to reflect a gradual resolution of uncertainty, all other stochastic parameters are assumed to be independent. Joint return effects are defined according to the description in Section 4.2.1. Several implementations with varying sample sizes and number of replications are displayed in Tables 7 and 8. The number preceding the letter S in the table notation represents the number of samples, while the number preceding the letter R is the number of replications.

Computations were performed on a PC with an Intel Core 2 Duo 2.0 GHz processor and 2GB of internal memory, using ILOG CPLEX Version 10.0. Although the computational studies were conducted on a single computer, the proposed solution procedure can easily be parallelized by solving the scenario subproblems on multiple machines to improve the solution times significantly.

The first two columns after the problem size information in Tables 7 and 8 display the time in seconds per replication of the SAA implementation and the expected value estimation for a given solution, respectively. The next column is the average duality gap, which is an average of the gap over all replications. The adjusted optimality gap estimate is given in the last column, and is calculated according to (80), based on the best solution obtained using the developed procedure. The sample size N' to estimate the corresponding objective value of a candidate solution was selected as 100 and 50 for 5T and 10T implementations. As it is shown in these results tables, the calculations of the objective values when the first stage decisions are fixed can be performed significantly faster then the solution of the SAA problem. Table 9 displays the first stage solutions for all tested configurations of the SAA algorithm. In most cases, different configurations return the same solution, based on the methodology used to select the best solution among the candidate solutions. Furthermore, for these instances, the two stage and multi-stage solutions are not significantly different than each other.

Overall, the computational results show that the developed procedure is effective and efficient in solving the project portfolio optimization problem, which is a difficult multistage stochastic program with endogenous uncertainty. Even without the implementation of a branch and bound procedure to close the duality gap, obtained lower bounds are very close to the Lagrangian upper bounds. As expected, the duality gap is less in instances with small sample sizes, while the optimality gap estimate is very low for large sample sizes. For the latter case, the variances are much lower and convergence of $\bar{v}_N^M$ and $\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)$ occur significantly faster, in the expense of slower computation times. The selection of the best solution out of several SAA solutions was done in two steps. In the first step, candidate solutions were identified based on the frequency of occurrences in the SAA solutions. Then the expected returns were estimated for these candidate solutions as described above, and the solution with the highest expected return estimate was selected. In Figure 14, we show the different levels of variance and convergence in this process on the 5T instances for both $MPPM$ and $2PPM$. The horizontal line in each plot represents the value of the estimate $\bar{v}_N^M$ for the corresponding algorithmic configuration. The effects of large sample sizes are evident in these plots, as it can be seen that convergence to the corresponding objective value is much faster in these cases. In addition, when compared with the two-stage model, convergence is better in the multi-stage case, mainly due the flexibility in a multistage model in rebalancing the portfolio in later stages. Hence, the results for different scenarios do not vary significantly.

Table 6: Data for the ten project test instance of stochastic project portfolio optimization problem

Attributes / Projects	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Fixed activity cost (mil.$)	0.2	0.1	0.3	0.2	0.2	0.05	0.1	0.2	0.05	0.3
Min. req. inv.(mil.$) / probability	2 / 0.35	3 / 0.3	4 / 0.5	2 / 0.4	1 / 0.5	1 / 0.6	5 / 0.5	1 / 0.5	2 / 0.55	1 / 0.3
Max. req. inv.(mil.$) / probability	4 / 0.65	5 / 0.7	6 / 0.5	6 / 0.6	3 / 0.5	3 / 0.4	7 / 0.5	1 / 0.5	4 / 0.45	3 / 0.7
Implementation time (yrs)	5	2	3	3	2	3	1	4	2	2
Min. initial return est. (mil.$) / prob.	1.5 / 0.6	1 / 0.4	1.5 / 0.14	0 / 0.225	0 / 0.5	2 / 0.25	3 / 0.83	2 / 0.67	1.5 / 0.5	1 / 0.5
Max. initial return est. (mil.$) / prob.	4.5 / 0.4	3.5 / 0.6	4.5 / 0.86	3.5 / 0.775	1 / 0.5	5 / 0.75	7 / 0.17	3 / 0.33	4/ 0.5	4.5 / 0.5
Prob. of min. return after low initial real.	*0.8*	*0.8*	*0.8*	*0.6*	*0.5*	*0.7*	*0.5*	*0.4*	*0.7*	*0.6*
Prob. of max. return after low initial real.	*0.2*	*0.2*	*0.2*	*0.4*	*0.5*	*0.3*	*0.5*	*0.6*	*0.3*	*0.4*
Prob. of min. return after high initial real.	*0.3*	*0.3*	*0.3*	*0.2*	*0.5*	*0.3*	*0.2*	*0.1*	*0.5*	*0.4*
Prob. of max. return after high initial real.	*0.7*	*0.7*	*0.7*	*0.8*	*0.5*	*0.7*	*0.8*	*0.9*	*0.5*	*0.6*
Dependent	-	C	B	E	D	-	J	-	-	G
Joint effect after min-min return (mil.$)	-	-0.5	-0.5	0	0	-	0.5	-	-	0.5
Joint effect after min-max return (mil.$)	-	-1.5	-2	0	0	-	-0.5	-	-	0
Joint effect after max-max return (mil.$)	-	-3	-3	-0.5	-0.5	-	-3	-	-	-3

Table 7: Computational Results for $MPPM$ - * and ** indicate that the best solutions were the same

Instance	app. #of rows	app. #of columns	sec/rep	sec/rep (soln.)	avg. duality gap (%)	$\bar{v}_N^M$	$\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)$	$\hat{\sigma}^2_{\bar{v}_N^M}$	$\hat{\sigma}^2_{\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)}$	adj.opt.gap est.
5T5S100R-MPPM	4,500	3,000	572	5.69	1.189	82.009	81.583	0.889	0.672	2.875
5T10S50R-MPPM	11,000	5,500	780	6.85	1.170	80.695	81.745	0.956	0.398	1.231 *
5T25S20R-MPPM	45,000	13,500	1398	23.09	2.438	80.373	81.117	0.731	0.217	1.164 *
5T50S10R-MPPM	146,000	27,000	3552	77.52	4.408	78.583	80.421	0.889	0.073	0.084 *
10T5S100R-MPPM	8,500	6,000	1706	84.32	1.020	228.596	230.736	5.851	2.090	3.383
10T10S50R-MPPM	22,000	11,000	1860	122.34	1.280	229.444	232.035	2.836	5.580	3.095 **
10T25S20R-MPPM	88,000	27,000	3480	204.10	1.870	229.033	232.314	2.034	1.754	0.534 **
10T50S10R-MPPM	290,000	54,000	3552	652.45	2.438	225.321	230.022	9.226	1.365	1.678 **

Table 8: Computational Results for $2PPM$ - * and ** indicate that the best solutions were the same

Instance	app. #of rows	app. #of columns	sec/rep	sec/rep (soln.)	avg. duality gap (%)	$\bar{v}_N^M$	$\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)$	$\hat{\sigma}^2_{\bar{v}_N^M}$	$\hat{\sigma}^2_{\hat{g}_{N'}(\hat{\mathbf{x}}_N^m)}$	adj.opt.gap est.
5T5S100R-2PPM	2,500	1,500	58	2.28	0.769	82.230	83.210	0.833	0.774	1.505 *
5T10S50R-2PPM	4,500	3,000	69	4.50	1.753	80.808	81.701	0.741	0.346	1.150 *
5T25S20R-2PPM	11,500	7,000	168	12.01	2.770	79.900	81.356	0.617	0.185	0.299 *
5T50S10R-2PPM	23,000	14,000	204	28.05	3.060	79.394	81.650	1.582	0.104	0.289 *
10T5S100R-2PPM	4,500	3,000	795	33.27	0.586	232.231	232.282	3.613	3.152	5.047 **
10T10S50R-2PPM	8,500	6,000	992	39.60	0.894	232.679	232.793	2.458	2.483	4.243 **
10T25S20R-2PPM	21,500	14,000	1446	48.12	1.709	228.650	230.204	1.747	2.564	2.516
10T50S10R-2PPM	43,000	28,000	2040	66.70	2.625	228.610	231.917	1.268	1.804	0.128 **

Table 9: First period solutions for different configurations of the SAA algorithm for the ten project test instance of stochastic project portfolio optimization problem

Project / SAA Configuration	**10T5S100R-MPPM**	**10T10S50R-MPPM**	**10T25S20R-MPPM**	**10T50S10R-MPPM**	**10T5S100R-2PPM**	**10T10S50R-2PPM**	**10T25S20R-2PPM**	**10T50S10R-2PPM**
A								
B								
C								
D								
E								
F	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
G	0.75				0.75	0.75		0.75
H	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
I							0.75	
J		0.75	0.75	0.75				
First Period Budget	3	3	3	3	3	3	3	3
First Period Investment	3	3	3	3	3	3	3	3

4.2.5 Stochastic Model with Resource Usage Minimization and Implementation Deadlines

As discussed in Section 4.1.2, another problem of interest, especially for capital investment portfolios, is the trade-off between the invested amount and the realized return for the portfolio. Furthermore, if there are certain deadlines by which the projects must be completed, these must be taken into account in determining the best project portfolio.

For the stochastic version of this multiobjective problem, a similar formulation to the deterministic model can be devised. However, a complicating factor in the stochastic case is that the calculated budget should produce feasible results for all (or most) realizations of the uncertain parameters. This issue must especially be addressed, if the problem is formulated as a two-stage or multi-stage problem where future budget levels are treated as first-stage variables. Another disadvantage of a formulation that is similar to the one presented for the deterministic case is due to the

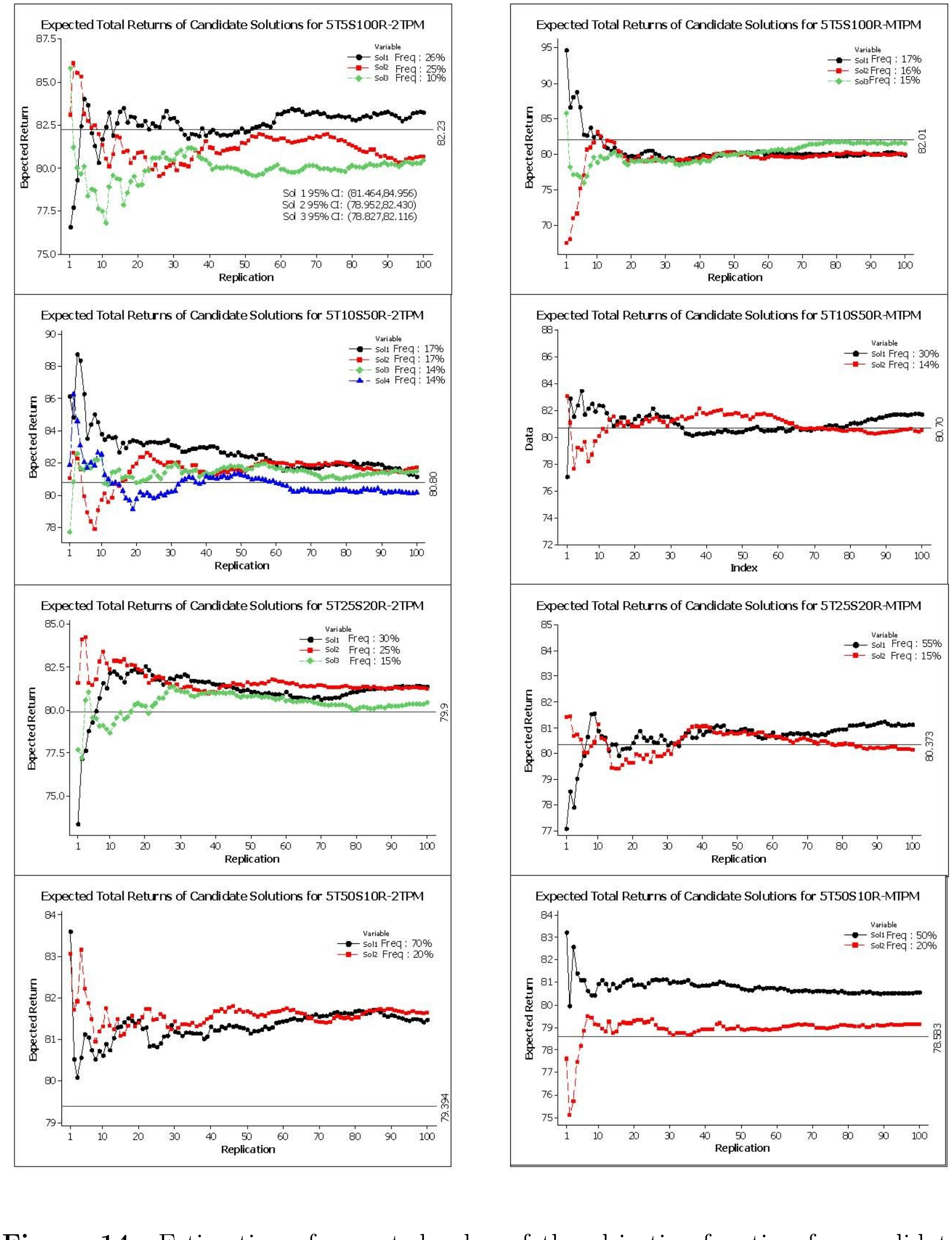

Figure 14: Estimation of expected value of the objective function for candidate solutions using samples sizes of $N' = 100$

difficulty of determining the correct weight and scaling factors for the two conflicting objectives. These factors can be different for different problems, and require a high level of involvement by the decision maker.

Hence, a sequential consideration of the objectives is a more suitable approach for this type of multiobjective problems, which can be used to determine a set of Pareto optimal solutions to be presented to the decision maker for final decision. The procedure is as simple as solving the problem multiple times for different budget levels. However, for efficiency, a systematic approach can be described as follows. In the following procedure, we assume that each project has a deadline. For those projects without an implementation deadline, a fake deadline can be established at the end of the planning period without any produced return, if the project is not implemented before the fake deadline.

Algorithm 4 (Procedure to Determine Pareto Optimal Solutions for the Stochastic Project Portfolio Optimization Problem with Resource Usage Minimization and Implementation Deadlines)**.** The steps of the procedure are as follows:

Step 1. Solve the stochastic problem with resource minimization objective and implementation deadline constraints, and determine minimum resource levels required to meet deadlines for all scenarios. Calculate the corresponding total return. Note that time value of the resources can be accounted for by expressing the objective function using discount terms.

Step 2. For return values greater than the return calculated in Step 1, solve the problem in Step 1 by adding another constraint so that the return level is greater than the considered value.

Step 3. Repeat Step 2 until the best return value is determined, which can be calculated by solving the stochastic program with return maximization objective and implementation deadline constraints.

The suggested approach is a powerful tool that provides an overall picture of the

Table 10: Pareto optimal solutions for a multiobjective stochastic project portfolio test problem

Required Resource (mil.$)	25.7	25.75	25.85	25.95	26	26.3	26.5	26.61	26.75
Total Return (mil.$)	111.53	111.55	112.47	112.8	115	115.31	115.81	116.16	116.16

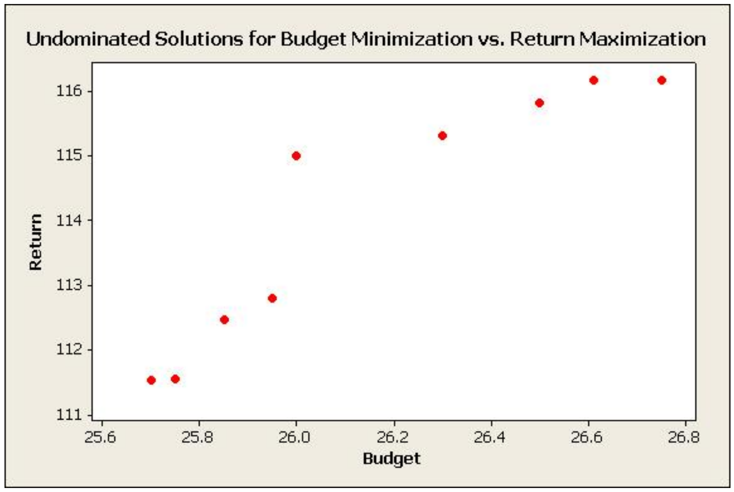

Figure 15: Graphical representation of Pareto optimal solutions for a multiobjective stochastic project portfolio test problem

trade-off relations between the two conflicting objectives of resource minimization versus return maximization. However, there are three disadvantages of this approach. First, it requires the solution of several large-scale problems, which may lead to very long solution times for some instances of the problem. Secondly, the procedure does not allow prioritization of the planning periods in determining the resource levels for a period. Hence, the minimization is performed for the overall resource requirement of the portfolio. However, this disadvantage can be overcome by introducing weights for the resources in each period in the resource minimization objective. Finally, as seen in Figure 15 for an example problem, the trade-off curve does not form a smooth concave curve due to the integralities in the problem formulation. Hence, for exact evaluation of a point on the curve, a separate problem has to be solved, rather than interpolating based on a fitted curve. In Table 10, we display the quantitative trade-off between the two objectives for an example problem. The same information is shown graphically in Figure 15, displaying the non-smoothness of the Pareto curve.

CHAPTER V

CONCLUSION

In this study, we have described two classes of multistage stochastic programming problems, and discussed the shortcomings in the literature for these classes of problems. We then developed efficient formulations and solution procedures for these problems, as remedies to the shortcomings identified. In this chapter, we describe the practical and theoretical conclusions and contributions of the study, as well as possible extensions to the work performed.

5.1 Conclusions for the Stochastic Network Capacity Planning Problem

Analysis of the modeling and solution techniques developed for the stochastic network capacity problems with different number of stages suggest that the proposed model is an innovative and powerful tool that can be used in capacity planning in networks. For given resource limitations and other feasibility constraints, the solutions provide an optimal expansion policy for a network such as an airport terminal, where the resulting configurations account for the uncertainty of future demand and minimize the expected amount of congestion over the planning horizon. For example, for the test problem with the large network in Figure 5, which is based on the South Terminal at Hartsfield-Jackson Atlanta International Airport, the solution provides information on how much capacity addition must be made in each area of the terminal during multiple expansion periods so that the overall congestion is minimized. This is a significantly improved approach, since current practice is to consider each area individually for any type of expansion.

To the best of our knowledge, our model is the first holistic capacity model for network structures. Specifically for airport terminal planning, given the inefficiency and congestion associated with current airport designs, it is essential that accurate capacity planning is performed for new airport designs using concepts such as those that have been proposed in this study. The results of the study are also applicable to existing airports, as noted above for the South Terminal at Hartsfield-Jackson Atlanta International Airport. Current configurations of these airports can be modified according to the results to maximize efficiency. By adding feasibility constraints, the model can be configured to determine whether it is optimal to build a new terminal building rather than expanding the existing one. In all cases, by minimizing the need for expansion and optimizing expansion schedules for airports, significant cost reductions can be achieved. Similar arguments are also valid for other flow networks, including other types of passenger terminals, traffic networks or queuing systems.

In the theoretical context, the peak period approximations derived for airport terminals are applicable to any traffic flow network or queuing system where steady state is not attained and transient analysis is intractable. Furthermore, the upper bounding heuristic that has been developed can be used on other capacity expansion models in the literature.

The described model assumes that a layout structure for the studied network is available. However, in certain applications such as in the case of a new terminal design, an exact layout may not be known a priori. For these problems, assuming that a general architectural concept has been developed, a layout optimization method can be used to determine an initial design layout based on assumed transportation costs. The field of facility layout planning is well developed, and several models exist that optimize the layout of a facility given flow demand levels between different units of the facility (Heragu, 2006). Passageways and other transportation related components can then be superimposed on this initial layout to determine the layout structure necessary for the implementation of the devised capacity planning model.

Given limited resources and realized demand levels, the solution of the corresponding network capacity problem will provide optimum capacity allocations for each component of the studied layout. The method also allows the modeling of any layout related constraints, such as maximum possible expansion levels.

In addition to regular capacity expansion planning under regular operating conditions, the capacity model developed in this study can also be used for robust planning purposes for non-standard operating conditions, such as in the case of an interruption in the components of a network. One good example is passenger flow planning in an airport terminal during a containment procedure after a security breach. This is a relatively common occurrence in most major airports, especially after increased security alertness in airport terminals in recent years. One such incident was experienced at Hartsfield-Jackson Atlanta International Airport recently, when a passenger's misbehavior led to the evacuation of the whole terminal. However, the rescreening and containment process resulted with a huge amount of congestion and confusion in the terminal. To minimize the impact of these incidents on passenger flow, the proposed methodology can be used for evacuation or containment planning purposes. Several scenarios that capture the congestion effects in case of the unavailability of certain passageways or service stations can be modeled and analyzed using the developed model. Furthermore, for network systems where such interruptions are more frequent, the probability distributions of the occurrence of these incidents can be taken into account by integrating them into the stochastic programming model, when determining the overall capacity design and expansion plan for the system.

5.1.1 Possible Extensions to the Stochastic Network Capacity Planning Problem

Theoretical extensions to the results on network capacity planning include the comparison of the developed capacity planning heuristic with heuristics suggested for other capacity planning models. In addition, more efficient solution procedures can be developed either by improving the proposed method or through new approaches that would deal with the highly nonlinear and nonconvex structures in the stochastic approximations.

Several practical extensions to the study are also possible. These include the modification of the model to incorporate the uncertainties associated with expansion costs and resource levels. For airport planning, another extension is possible by integrating the model with the airside of an airport, thus forming a global capacity planning model for airports, and possibly for the whole national airspace system. Also, the problem can be modified slightly to handle different objectives and constraints. Some examples include cost minimization with level of service constraints, level of service maximization with resource constraints and different level of service criteria, and robust optimization of cost or level of service with disruption probabilities.

5.2 Conclusions for the Project Portfolio Optimization Problem

Project portfolio optimization problem has not been studied at the detailed level considered in this study before. It was also noted that the problem has a unique structure with endogenous uncertainty of the stochastic parameters, and development of a solution methodology would also contribute to the general class of such problems. We have presented a detailed and comprehensive description of the problem, the solution characteristics, and an efficient solution approach that can be used to solve this large-scale problem.

Implementation of the proposed models in project portfolio selection by organizations will lead to significant increases in returns, as all relevant inputs and uncertainty are captured in the models, as opposed to existing project portfolio selection tools. A significant contribution of the developed models is that they include a common but less studied characteristic of endogenous uncertainty. Problems of this type are usually difficult to model, since the nonanticipativity conditions require comparisons of scenario pairs. We present a compact decomposable structure which can be exploited by methods that are commonly used in the solution of classical stochastic programming problems. It must be noted that even if the endogenous uncertainty were to be ignored, the resulting problem would be a multistage stochastic integer program with several stages for which no general solution procedures are available. Hence, to handle the difficulty, an effective lower bounding algorithm and performance bounds have been developed as a part of the overall solution procedure. The algorithm has been tested with promising results, and it is believed that such a procedure can be implemented in several other similar problems.

In summary, the project portfolio optimization problem is a difficult practical problem, for which a comprehensive model and solution methodology has not been developed in the existing limited approaches in the literature. In this study, we fill this gap by formally defining and effectively modeling several complexities that are inherent in this problem, and developing efficient solution procedures. More specifically, contributions of this study include the following: A comprehensive model that captures all relevant concerns in project portfolio management has been developed. To the best of our knowledge, it is the first such approach that (i) provides an accurate representation of the stochastic decision process in project portfolio management, (ii) models the endogenous uncertainty inherent in this decision process, and at the same time (iii) includes a computationally practical solution procedure. In addition, from a theoretical standpoint, contributions are as follows: (i) a new and efficient formulation technique to model nonanticipativity in multistage stochastic programs with endogenous uncertainty is developed, (ii) the developed formulation enables scenario based decomposition in such problems, in addition to the application of other methods developed for classical multistage stochastic programs, and (iii) a tight lower bounding algorithm based on feasible dual conversion that can be used for any similarly structured problem is developed.

5.2.1 Possible Extensions to the Project Portfolio Optimization Problem

Extensions of the study are possible in several areas. Integration of risk is an important part of the technology portfolio selection, since most practical decisions are made while considering risks associated with the investment decisions. A similar extension is also possible for the case with resource usage minimization objective. A better modeling of this version of the problem can be studied according to the practical needs of the decision makers. One other extension includes capturing the effects of dependencies in probability distributions on the investment decisions.

Another solution approach, which is worth further investigation and also takes into account the risk factors through a variance measure, can also be studied for the project portfolio optimization problem. Consider the candidate solutions obtained through a sampling procedure, and the values of the first stage investment variables and their corresponding objective function values in $R^{|\mathcal{N}|+1}$. Our goal is to identify ranges of values for x_{i1} in this vector space such that the mean objective function value in the range is maximum, while the variance is minimum. It is expected that these points in $|\mathcal{N}|+1$ dimensions will be dense in certain ranges. A search procedure can be implemented to identify these ranges. Once these ranges are identified, a heuristic method can be implemented to determine a feasible investment strategy for the current investment period. One such heuristic is to invest at the maximum allowable level for each technology in the order of expected performance levels. Another method could be to determine an investment policy in which the investment levels are as close as possible to the expected values of the variables. As a further research topic, such a procedure can be implemented and compared with the results of the models developed in this study.

APPENDIX A

MAXIMUM PEAK PERIOD DELAY APPROXIMATIONS IN PEDESTRIAN AND QUEUING NETWORKS

The goal of most terminal capacity analyses is to minimize congestion related passenger delay in the terminals. In addition, most flow networks contain processing stations with a queuing structure. Hence, approximation of walking times in passageways and delay times at processing stations as a function of capacity and flow rates is an important part of any capacity planning model. Due to the stochastic and transient nature of demand, most such estimations are based on observational data or simulation models, which do not provide appropriate inputs for optimization models. We consider the walking and processing delays separately, and develop delay time approximations for the two areas. These approximations make it possible for such effects to be considered in optimization models.

A.1 Maximum Delay in Passageways

The following approximation can be developed for maximum pedestrian delay in passageways of a terminal structure. These results are also discussed in Solak *et al.* (2006).

Approximation 1. *Maximum walking time in a passageway of length L and width w, when the peak flow rate is f, can be approximated by the following delay function.*

$$t^{\omega} = \frac{Lw}{-0.000188f + 1.34w} \tag{108}$$

where L and w are in meters, t^{ω} is in seconds, and f is in passengers per hour.

Proof. Although there have been several studies on travel time functions for vehicular traffic and general pedestrian traffic (Older, 1968; Fruin, 1971; Tanariboon *et al.*, 1986; Virkler & Elayadath, 1994; Sarkar & Janardhan, 1997), such studies are rare for pedestrians in transportation terminals. One exception is Young (1999), in which pedestrian walking speeds are observed and analyzed in two major airport terminals. Results from this study suggest that free-flow walking speeds in airport terminals are normally distributed with a mean of 80.5 m (264 ft) per minute and a standard deviation of 15.9 m (52 ft) per minute. Regardless of domain, all of the pedestrian traffic studies include estimations of the relationship between the speed of pedestrians and the congestion levels. Using the free-flow speeds from Young (1999) to adjust the relationship suggested by Sarkar & Janardhan (1997), we derive the following linear function to represent the relation between the speed (m/s) and density (passengers/m^2) in airport terminal passageways:

$$s = -0.34\phi + 1.34 \tag{109}$$

where s represents the speed and ϕ is the density. To approximate the maximum walking time in a passageway l_ω of length L, maximum density in the passageway can be estimated using the peak flow rate f and width w of the passageway. Assuming that the peak load is instantenous and that interarrival times $I = 1/f$ are exponentially distributed, the mean and variance of the number of passengers in the passageway, N, can be obtained using the following second order approximations based upon truncated Taylor series expansions (Rice, 1995):

$$\begin{aligned} E[N] &= E\Big[\frac{t_o}{I}\Big] \\ &= \frac{E[t_o]}{E[I]}\Big(1 + \frac{Var[I]}{E^2[I]}\Big) \end{aligned} \tag{110}$$

$$\begin{aligned} Var[N] &= Var\Big[\frac{t_o}{I}\Big] \\ &= \Big(\frac{E[t_o]}{E[I]}\Big)^2\Big(\frac{Var[t_o]}{E^2[t_o]} + \frac{Var[I]}{E^2[I]} - \frac{Var^2[I]}{E^4[I]}\Big) \end{aligned} \tag{111}$$

where the random variable $t_o = \frac{L}{s_o}$ is the walking time under free-flow conditions. $E[t_o]$ and $Var[t_o]$ can be estimated using similar approximations, i.e.

$$\begin{aligned} E[t_o] &= \frac{L}{E[s_o]}(1 + \frac{Var[s_o]}{E^2[s_0]}) \\ &= \frac{L}{80.5}(1 + \frac{15.9^2}{80.5^2}) = 0.0130L \end{aligned} \tag{112}$$

$$\begin{aligned} Var[t_o] &= (\frac{L}{E[s_o]})^2(\frac{Var[s_o]}{E^2[s_o]} - \frac{Var^2[s_o]}{E^4[s_o]}) \\ &= (\frac{L}{80.5})^2(\frac{15.9^2}{80.5^2} - \frac{15.9^4}{80.5^4}) = 0.0024^2L^2 \end{aligned} \tag{113}$$

It follows from (110) and (111) that

$$\begin{aligned} E[N] &= \frac{0.0130Lf}{60}(1+1) \\ &= 0.000433Lf \end{aligned} \tag{114}$$

$$\begin{aligned} Var[N] &= (\frac{0.0130Lf}{60})^2(\frac{0.0024^2L^2}{0.0130^2L^2} + 1 - 1) \\ &= 0.00004^2L^2f^2 \end{aligned} \tag{115}$$

We assume that the distribution of N is normal, and propose the following design density $\tilde{\phi}$ for passageway l_ω:

$$\tilde{\phi} = \frac{E[N] + 3\sigma_N}{A} = \frac{0.000553f}{w} \tag{116}$$

where $A = wL$ is the total effective area of the passageway. Hence, it follows from (109) that the maximum walking time in a passageway can be approximated by

$$t^\omega = \frac{Lw}{-0.000188f + 1.34w} \tag{117}$$

where L and w are in meters, t^ω is in seconds, and f is given in passengers per hour. $\square$

A.2 Maximum Delay in Processing Stations

Most congestion at airport terminals occurs at processing stations such as security checkpoints and check-in counters, where the demand is known to follow transient

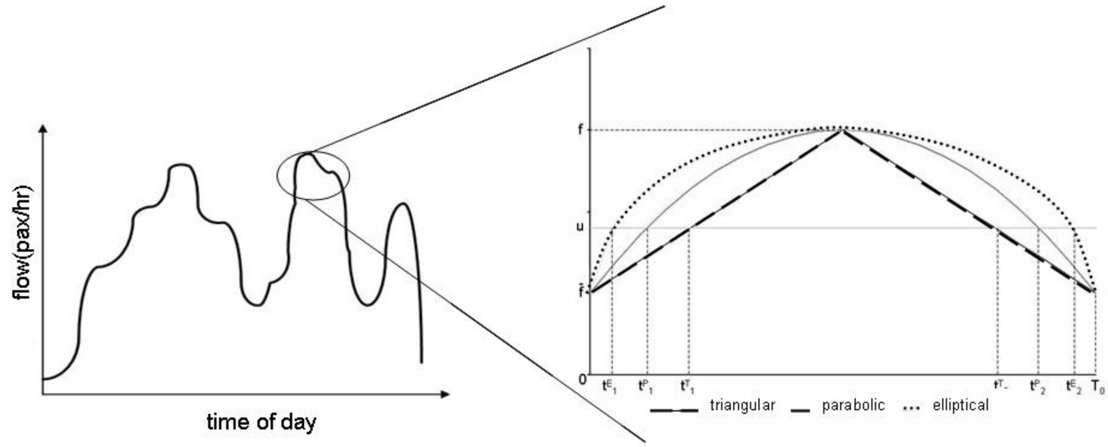

Figure 16: Highest peak is identified and approximated by a triangular, parabolic or half-elliptical function

patterns. Similarly, for any type of queuing network with transient demand levels, it is essential that some representation of queues is developed to serve as inputs in optimization problems studying these systems. Solak *et al.* (2006) develop relations to estimate the maximum delay at the processing stations queuing networks as a function of flow and capacity. A deterministic approach with varying arrival rates over time and constant process rates, and a stochastic extension are studied based on applications in airport terminals.

A.2.1 Deterministic Approximation

For an airport terminal, passenger arrival rates, estimated from flight schedules, can be plotted against time as shown in Figure 16. On this plot, the highest peak that can be identified is used in peak demand analysis for design purposes. A peak is defined as a period during which the arrival rate remains above the average arrival rate. Approximations can be used to represent the shape of a peak, and these approximations can be used further to estimate the maximum queue length. Depending on the sharpness of the peak, either a triangular, parabolic or half-elliptical approximation is considered as shown in Figure 16.

Approximation 2. *Assuming deterministic service times, maximum delay t^ρ at a process station for different peak types can be approximated as follows:*

$$t_T^\rho = \frac{(u-f)^2 T_o}{2cfu} \tag{118}$$

$$t_P^\rho = \frac{2T_o(f-u)^{3/2}}{3u\sqrt{cf}} \tag{119}$$

$$t_E^\rho = \frac{T_o \pi}{4cuf}(f-u)^{3/2}\sqrt{u-(1-2c)f} \tag{120}$$

where u is the capacity of the processing station, T_o is the time when the arrival rate drops below the average arrival rate $\bar{f}$, and $c = 1 - \frac{\bar{f}}{f}$ is a constant.

Proof. Triangular Peak Approximation

If $f(t)$ represents the flow rate into a processing station l_ρ over time, then the triangular peak function can be expressed as

$$f(t) = \begin{cases} \bar{f} + a_T t & 0 \le t \le T_o/2 \\ \bar{f} + a_T(T_o - t) & T_o/2 < t \le T_o \end{cases} \tag{121}$$

where T_o is the time when the arrival rate drops below the average arrival rate $\bar{f}$, and $a_T = \frac{2(f-\bar{f})}{T_o}$ with f representing the maximum flow rate. Assuming that queue buildup occurs only after the arrival rate exceeds the capacity u of the station, the maximum queue length can be estimated by calculating the area between the capacity line and the triangular arrival rate curve in Figure 16. This area is equal to $\frac{(t_2-t_1)(f-u)}{2}$, where t_1 and t_2 represent the times when the arrival rate is equal to the capacity. These values can be obtained from the following relation:

$$u = \bar{f} + a_T t_1 = \bar{f} + a_T(T_o - t_2) \tag{122}$$

It follows that the maximum queue length, Q_{max}, can be expressed as

$$Q_{max} = \frac{\left(T_o - 2\left(\frac{u-\bar{f}}{a_T}\right)\right)\left(f-u\right)}{2} \tag{123}$$

Then, assuming deterministic service times, maximum delay t_T^ρ at a process station with a triangular peak can be approximated by $\frac{Q_{max}}{u}$, giving the following result:

$$t_T^\rho = \frac{(u-f)^2 T_o}{2cfu} \tag{124}$$

In this expression, $c = 1 - \frac{\bar{f}}{f}$ is a constant.

Parabolic Peak Approximation

For a parabolic approximation of the peak, the arrival rate curve is

$$f(t) = f - a_P\big(t - \frac{T_o}{2}\big)^2 \tag{125}$$

where $a_P = \frac{4(f-\bar{f})}{T_o^2}$. Hence, t_1 and t_2 are the roots of the following polynomial function of the second degree:

$$f - a_P\big(t - \frac{T_o}{2}\big)^2 - u = t^2 - T_o t - \big(\frac{4u - 4f + a_P T_o^2}{4a_P}\big) = 0 \tag{126}$$

It follows that

$$\begin{aligned} t_2 - t_1 &= \frac{T_o + \sqrt{T_o^2 - \frac{4u-4f+a_P T_o^2}{a_P}}}{2} - \frac{T_o - \sqrt{T_o^2 - \frac{4u-4f+a_P T_o^2}{a_P}}}{2} \\ &= T_o\sqrt{\frac{f-u}{cf}} \end{aligned} \tag{127}$$

Similar to the triangular case, the area of the region between the capacity and parabolic arrival rate curves in Figure 16 represents the maximum queue length. Thus, maximum time spent for the deterministic parabolic approximation of the peak at process station l_ρ can be expressed as follows:

$$t_P^\rho = \frac{2T_o(f-u)^{3/2}}{3u\sqrt{cf}} \tag{128}$$

Half-elliptical Peak Approximation

Another approximation can be performed assuming that the peak has a half ellipsoid shape as shown in Figure 16. In this case, the arrival rate function can be expressed as

$$f(t) = \bar{f} + \sqrt{\big(f-\bar{f}\big)^2\big(1 - \frac{(2t-T_o)^2}{T_o^2}\big)} \tag{129}$$

It follows that the queue builds up for a period of

$$t_2 - t_1 = \frac{T_o}{cf}\sqrt{(f-u)(u-f+2cf)} \tag{130}$$

Thus, the maximum delay can be estimated as

$$t_E^{\rho} = \frac{T_o \pi}{4cuf}(f-u)^{3/2}\sqrt{u-(1-2c)f} \tag{131}$$

□

A.2.2 Stochastic Approximation

Although deterministic approximations of delay times provide a means to estimate the total passenger delay in an airport terminal, stochastic effects need to be studied to determine whether they would help provide a more accurate representation of passenger flow. The following approximations account for the stochasticity of service times.

Approximation 3. *For the stochastic case, delay times can be approximated as follows:*

$$t_{TS}^{\rho} = \frac{\tilde{Q}_{max}}{u} = \frac{0.95\sqrt[3]{u} + \frac{(u-f)^2 T_o}{2cf} + 3\sqrt{-0.3(\sqrt[3]{u})^2 + 2T_o(f-u) - \frac{3T_o(f-u)^2}{2cf}}}{u} \tag{132}$$

$$t_{PS}^{\rho} = \frac{0.95\sqrt[3]{u} + \frac{2T_o}{3\sqrt{cf}}(f-u)^{3/2} + 3\sqrt{-0.3(\sqrt[3]{u})^2 + 2T_o\sqrt{cf(f-u)} - \frac{4T_o(f-u)^{3/2}}{3\sqrt{cf}}}}{u} \tag{133}$$

$$t_{ES}^{\rho} = \frac{0.95\sqrt[3]{u} + \frac{T_o\pi}{4cf}(f-u)^{3/2}\sqrt{u-(1-2c)f} + 3\sqrt{-0.3(\sqrt[3]{u})^2}}{u} \tag{134}$$

Proof. For the stochastic case, delay times can be approximated based on the estimation of the queue length distribution over time. An approach to this problem is suggested by Newell (1982), in which the distribution of the queue length during peak periods is studied using diffusion equations. More specifically, Newell (1982) suggests that the distribution of $Q(t)$ during peak periods is normal with the following mean and variance, provided that the queue is unlikely to vanish in a short period of time:

$$E[Q(t)] = 0.95\sqrt[3]{u} + \int_0^t [f(t') - u]dt' \tag{135}$$

$$Var[Q(t)] = -0.3(\sqrt[3]{u})^2 + \int_0^t [C_a f(t') + C_s u]dt' \tag{136}$$

where C_a and C_s are the squared coefficients of variation for the interarrival and service times, respectively. These results can be seen as stochastic corrections for the deterministic approximations discussed in Section A.2.1. If $t_2 - t_1$ represents the time period during which the arrival rate stays above the process rate, Newell (1982) shows that the relation $t_2 - t_1 \geq \frac{2}{(C_a+C_s)\sqrt[3]{u}}$ must hold for the approximations to be valid. For most practical applications, it is reasonable to assume that both interarrival and service times are exponentially distributed, which implies $C_a = C_s = 1$. Hence, for the validity of the approximations under these assumptions, it is required that $t_2 - t_1 \geq u^{-1/3}$.

We assume that above conditions hold, and develop expressions to approximate the expected value of the maximum delay for the triangular, parabolic and half-elliptical peak occurrences. However, the approach could be generalized to any shape that the peak can take. In situations where the peak does not resemble a geometric shape, piecewise calculations can be made to approximate the integrals in the expressions above. On the other hand, for an optimization model, it is necessary that a compact closed form expression be used to represent the delays in processing stations. Assuming that the queue length is distributed normally over time, the design criteria for the maximum queue length can be chosen as $\tilde{Q}_{max} = E[Q(t_2)] + 3\sigma_{Q(t_2)}$. Using (135) and (136), we calculate the expected value and the variance of the queue length for a triangular peak as follows:

$$\begin{aligned} E[Q(t_2)] &= 0.95\sqrt[3]{u} + \int_{t_1}^{T_o/2} [\bar{f} + a_T t - u]dt + \int_{T_o/2}^{t_2} [\bar{f} + a_T(T_o - t) - u]dt \\ &= 0.95\sqrt[3]{u} + \frac{(u-f)^2 T_o}{2cf} \end{aligned} \tag{137}$$

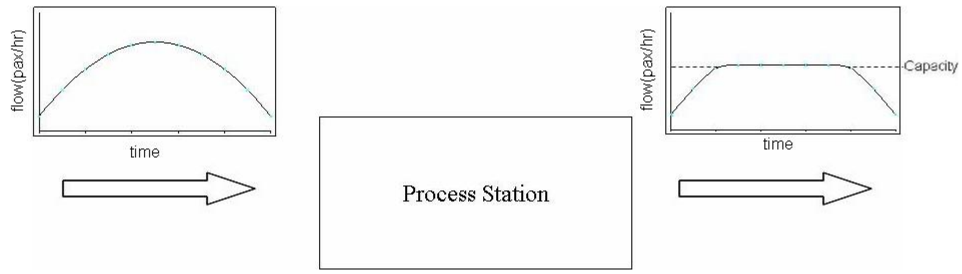

Figure 17: Effect of the capacity on the departure process from a service station

$$\begin{aligned}Var[Q(t_2)] &= -0.3(\sqrt[3]{u})^2 + \int_{t_1}^{T_o/2}[\bar{f} + a_T t + u]dt + \int_{T_o/2}^{t_2}[\bar{f} + a_T(T_o - t) + u]dt \\ &= -0.3(\sqrt[3]{u})^2 + 2T_o(f-u) - \frac{3T_o(f-u)^2}{2cf}\end{aligned} \tag{138}$$

It follows that,

$$t_{TS}^{\rho} = \frac{\tilde{Q}_{max}}{u} = \frac{0.95\sqrt[3]{u} + \frac{(u-f)^2 T_o}{2cf} + 3\sqrt{-0.3(\sqrt[3]{u})^2 + 2T_o(f-u) - \frac{3T_o(f-u)^2}{2cf}}}{u} \tag{139}$$

A similar analysis can also be performed for the parabolic and half elliptical peak cases, resulting with the above maximum delay functions. □

A.3 Approximations in Networks

The approximations above are valid when peak period analysis is performed on individual processing stations. However, in a network structure the propagation of demand has to be considered, as flow into downstream processes will be affected by the capacity of preceding processes. The effect of the capacity on the departure process from a service station is illustrated in Figure 17. As seen in this plot, the departure rate curve has a flat peak due to the capacity of the station. Hence, assuming the same peak shape for downstream processes could lead to an inaccurate estimation of the actual arrival pattern at these service stations. A better approximation can be obtained through the half-ellipse approximation. Hence, given any arrival curve at a station, the arrival rates for all downstream processes can be approximated by using the time functions obtained for the half-elliptical peak. However, in all cases, it is possible to analyze the functions individually, possibly through observations or simulation studies, and determine the best approximating shape.

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